Balancing Fairness and Efficiency in an Optimization Model

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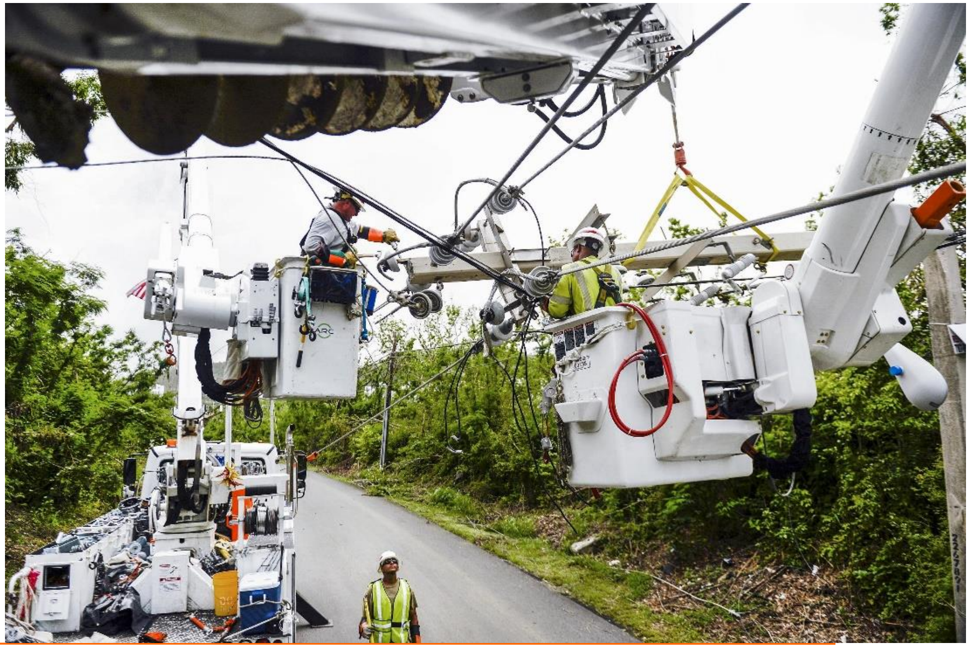
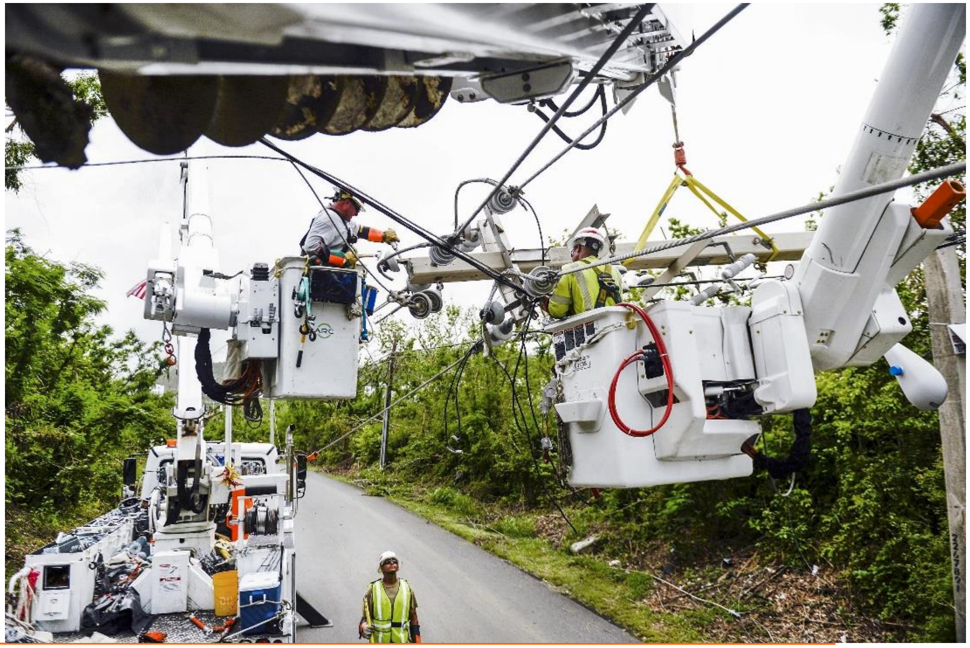
Efficiency vs. Fairness

- Example: disaster relief
  - Power restoration in Puerto Rico after Hurricane Maria can focus on San Juan and other **urban** areas first (**efficient solution**). **This leaves rural areas without power for weeks/months.**



Efficiency vs. Fairness

- Example: disaster relief
  - Power restoration in Puerto Rico after Hurricane Maria can focus on San Juan and other **urban** areas first (**efficient solution**). **This leaves rural areas without power for weeks/months.**
  - Or it can restore **rural** power as quickly as urban power (**fair solution**). **This delays power restoration for most people**, due to the difficulty of restoring rural power.



Efficiency vs. Fairness

- Other problem areas…
  - Health care resources.
  - Facility location (e.g., emergency services).
  - Taxation (revenue vs. progressivity).
  - Relief operations.
  - Telecommunications
  - Traffic signal timing

UNHCR
The UN Refugee Agency
Ύπατη Αρμοστεία του ΟΗΕ
για τους Πρόσφυγες

Utility vs. Equity

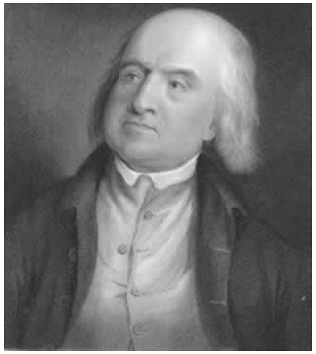
- Two classical criteria for distributive justice:
- **Utilitarianism** (max total benefit)

$$\max_x \sum_i u_i(x_i)$$

$$(x_1,\ldots,x_n) \in S$$

Utility function

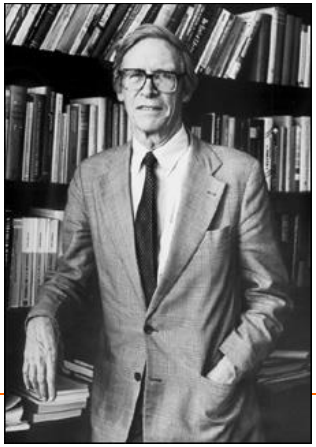
Resource allocation



- **Rawlsian difference principle = maximin** (max welfare of worst off)

$$\max_x \min_i \{u_i(x_i)\}$$

$$(x_1,\ldots,x_n) \in S$$



Combining Equity and Efficiency

- Find socially optimal distribution of utility by maximizing a social welfare function $F(\boldsymbol{u})$.
    - Problem: design a suitable SWF.

Combining Equity and Efficiency

- Find socially optimal distribution of utility by maximizing a social welfare function $F(\boldsymbol{u})$.
  - Problem: design a suitable SWF.
- Some well-known proposals:
  - **Alpha-fairness**
    - *Nonlinear. Choose α?*
  - **Proportional fairness**
    - α = 1. *Nash bargaining argument makes strong assumptions.*
  - **Kalai-Smorodinksy bargaining solution**
    - *Counterintuitive implications.*
  - **Convex combination of utility and maximin**
    - *How to choose weights?.*

$$F_\alpha(u) = \begin{cases} \dfrac{1}{1-\alpha} \sum_i u_i^{1-\alpha} & \text{for } \alpha \geq 0,\ \alpha \neq 1 \\ \sum_i \log(u_i) & \text{for } \alpha = 1 \end{cases}$$

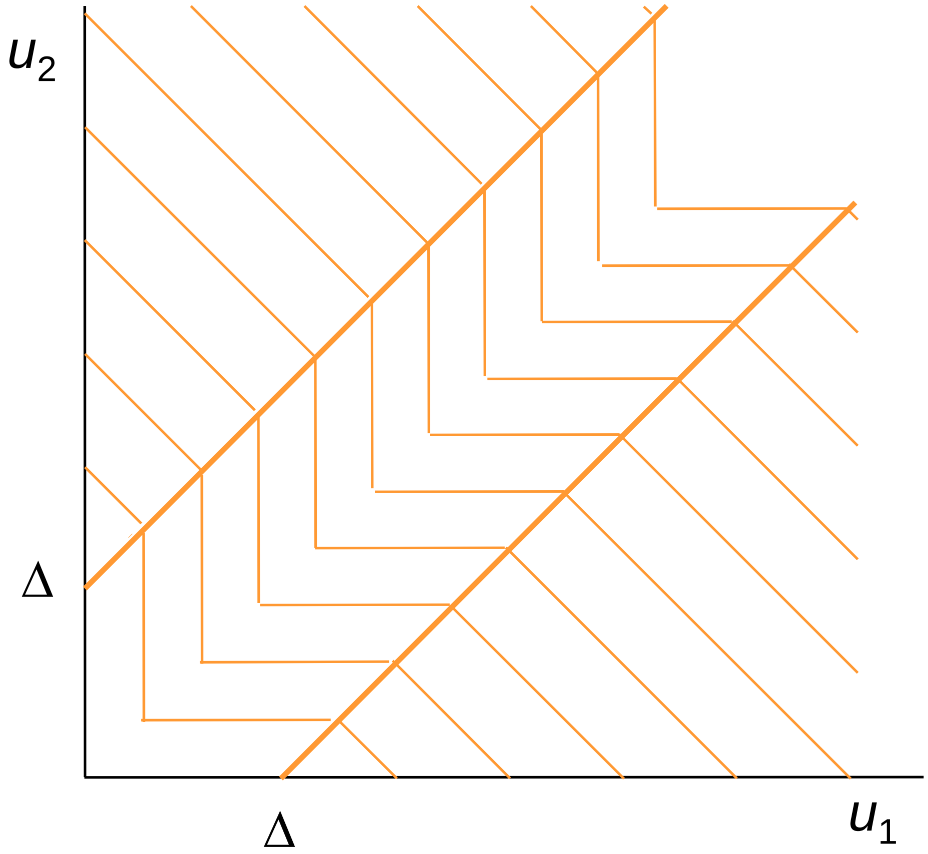
H-W Model

- Build on Hooker-Williams proposal (2012)
  - *Δ* regulates equity-efficiency tradeoff, has practical meaning
  - Build mixed integer programming model.
- For 2 persons:

$$\max\ F(u_1,u_2)=\begin{cases} u_1+u_2 & \text{if } \left|u_1-u_2\right|\geq\Delta \\ 2\min\left\{u_1,u_2\right\}+\Delta & \text{otherwise} \end{cases}$$

H-W Model

Contours of **social welfare function** for 2 persons.



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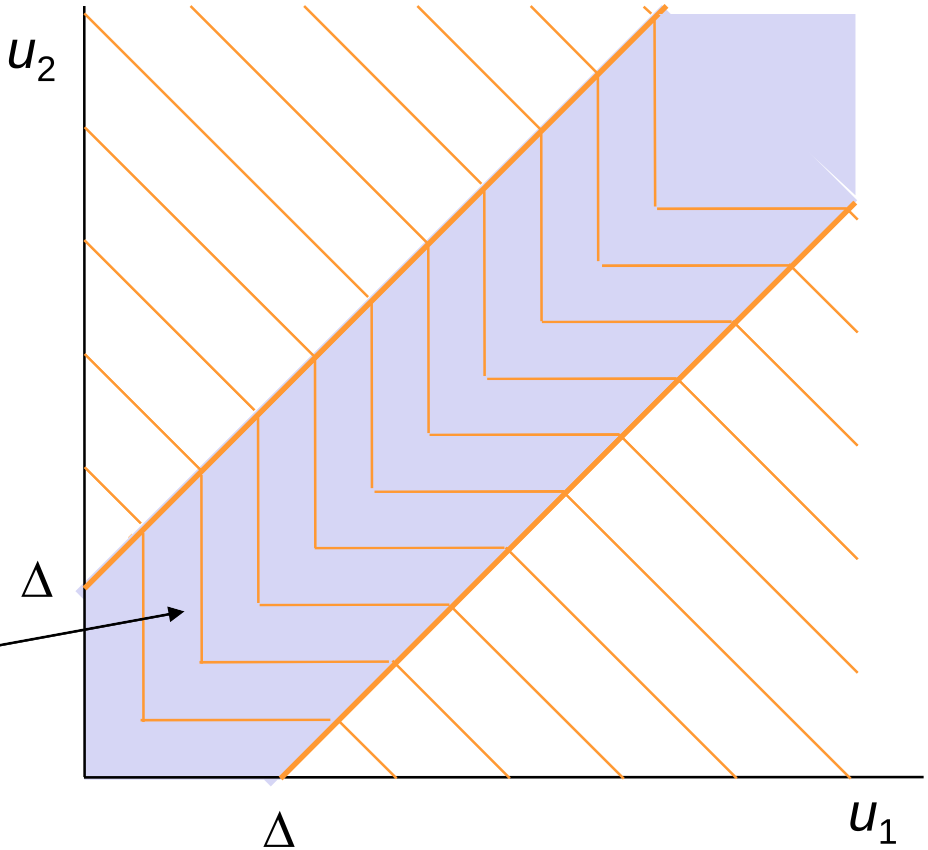


Maximin region

$2\min\{u_1, u_2\} + \Delta$

H-W Model

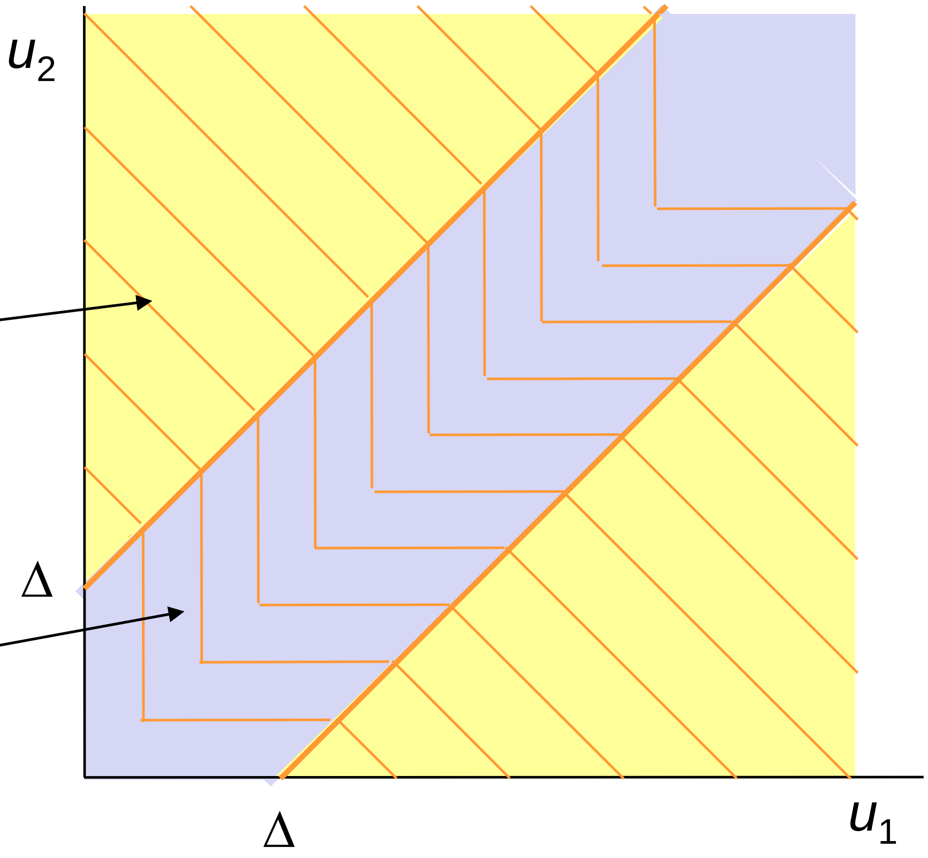
Contours of **social welfare function** for 2 persons.

Utilitarian region

$u_1 + u_2$

Maximin region

$2\min\{u_1, u_2\} + \Delta$



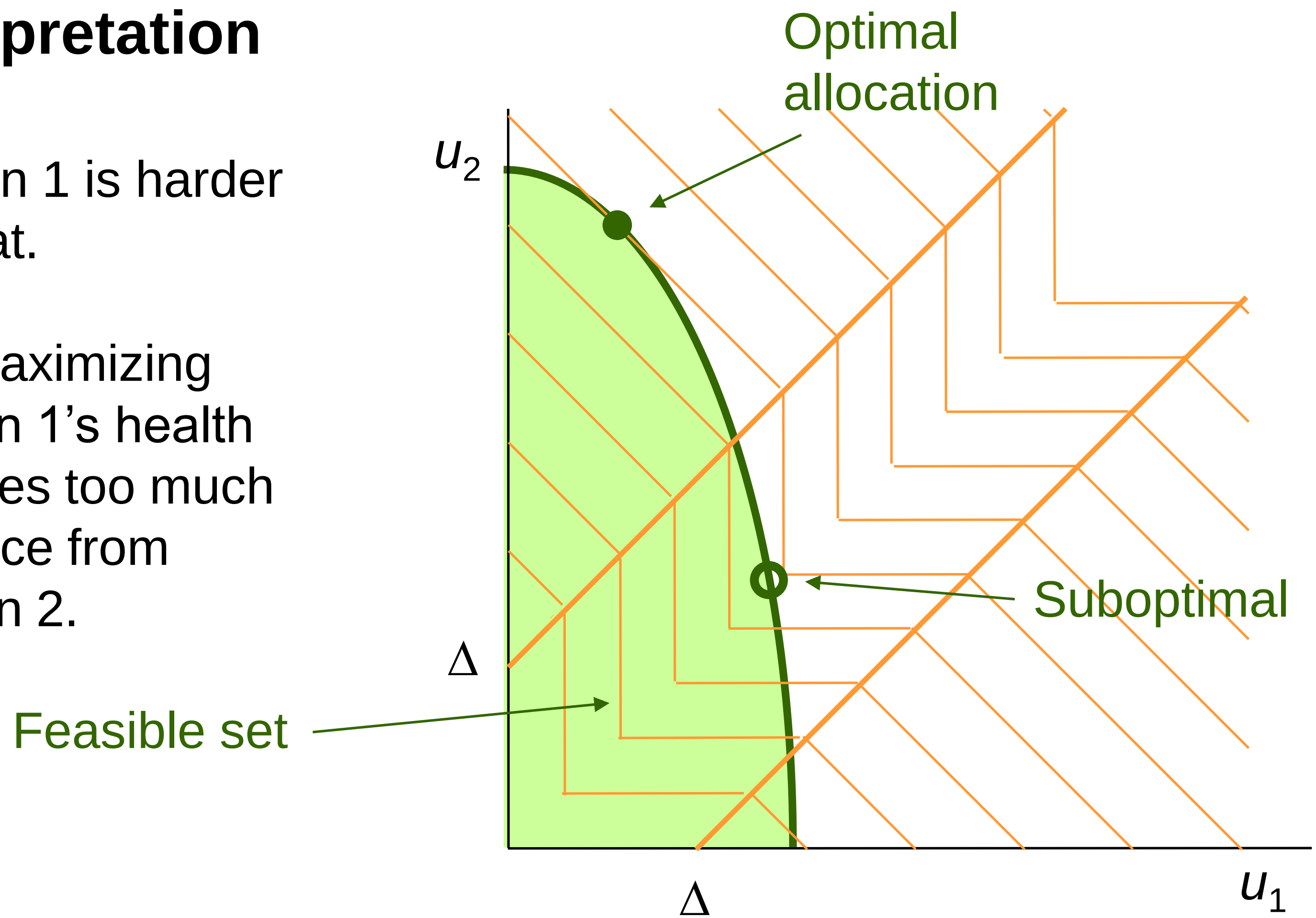
Ensures continuous contours

H-W Model

Healthcare interpretation

Person 1 is harder to treat.

But maximizing person 1's health requires too much sacrifice from person 2.



H-W Model

- ***n*-person social welfare function**

$$F(u) = (n-1)\Delta + n u_{\min} + \sum_{i=1}^{n} \left(u_i - u_{\min} - \Delta \right)^{+}$$

- Utilities in **fair region** (within Δ of smallest, $u_{\min}$) receive some **priority**.
    - That is, **disadvantaged** individuals receive some priority.
    - $\Delta = 0$: utilitarian SWF (no fair region)
    - $\Delta = \infty$: maximin SWF (all utilities in fair region)
    - Utilities in fair region are equated with smallest utility, which receives weight equal to number of utilities in fair region.

H-W Model

MILP model of H-W social welfare function:

$$\max \; z$$

$$z \le (n-1)\Delta + \sum_{i=1}^{n} v_i$$

$$u_i - \Delta \le v_i \le u_i - \Delta\delta_i, \;\; i = 1,\ldots,n$$

$$w \le v_i \le w + (M-\Delta)\delta_i, \;\; i = 1,\ldots,n$$

$$u_i \ge 0, \;\; \delta_i \in \{0,1\}, \;\; i = 1,\ldots,n$$

Assumes $u_i - u_j \le M$ for all i, j
to ensure MILP representability

Theorem. The model is correct and **sharp** (not easy to prove).

H-W Model

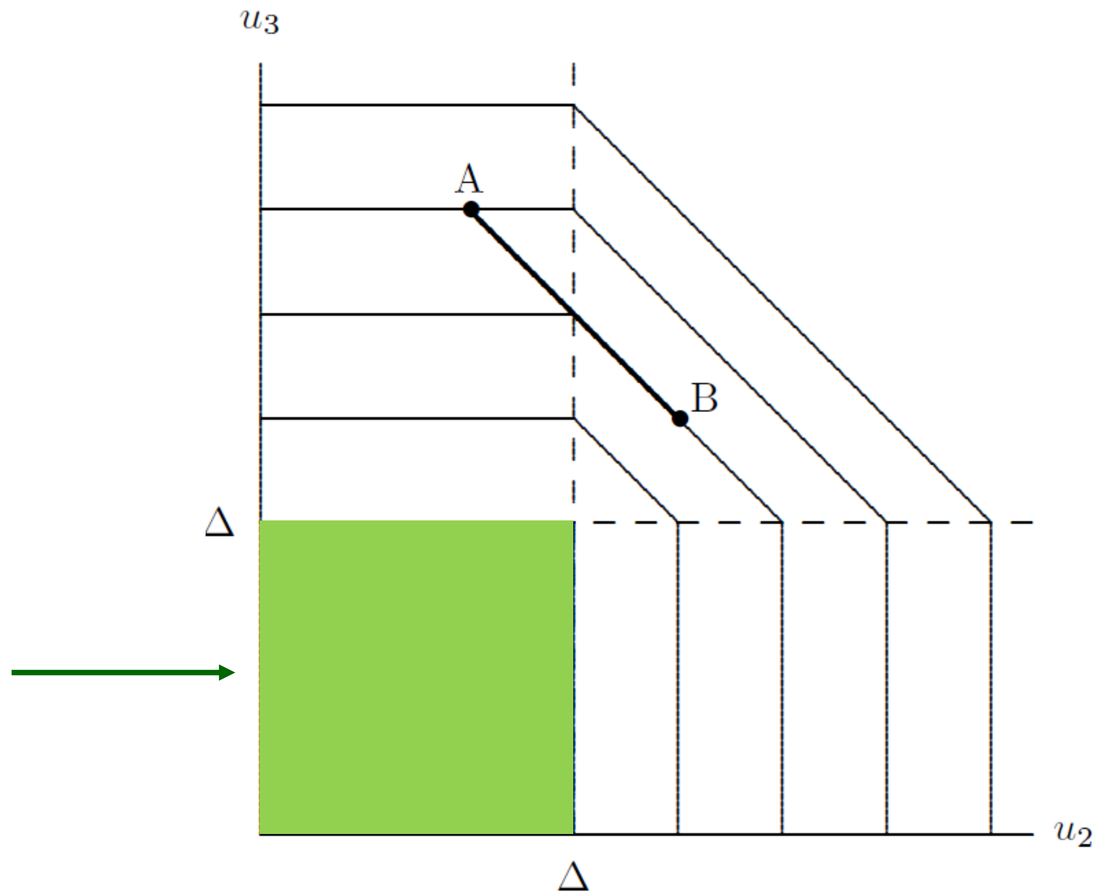
- Problem with H-W model
  - Utilities in fair region (other than u_{min}) **do not affect value of the social welfare function**.
  - There are many alternate socially optimal solutions with **very different equity properties**.

H-W Model

Example: 3 persons

Contours for $F(0,u_2,u_3)$

All solutions in this region have same social welfare value



Proposed Model

- Combine utilitarian and **leximax** criteria.

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  - Leximax: Let $u_{\langle i \rangle} = i$-th smallest utility.
  - Max $u_{\langle 1 \rangle}$ to obtain $\bar{u}_{\langle 1 \rangle}$, then max $u_{\langle 2 \rangle}$ with $u_{\langle 1 \rangle} = \bar{u}_{\langle 1 \rangle}$, etc.
  - Solve **sequence** of optimization problems.
  - Problem k determines $\bar{u}_{\langle k \rangle}$ while maximizing a social welfare function $F_k(u)$ that combines equity and utility.
  - $F_1(u)$ is H-W social welfare function.
  - $\bar{u}_{\langle k \rangle}$ receives weight $n-k+1$ in $F_k(u)$, larger $u_{\langle i \rangle}$s weight 1

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  - $\bar{u}_{\langle k \rangle}$ receives weight $n-k+1$ in $F_k(u)$, larger $u_{\langle i \rangle}$s weight 1
- New social welfare functions:

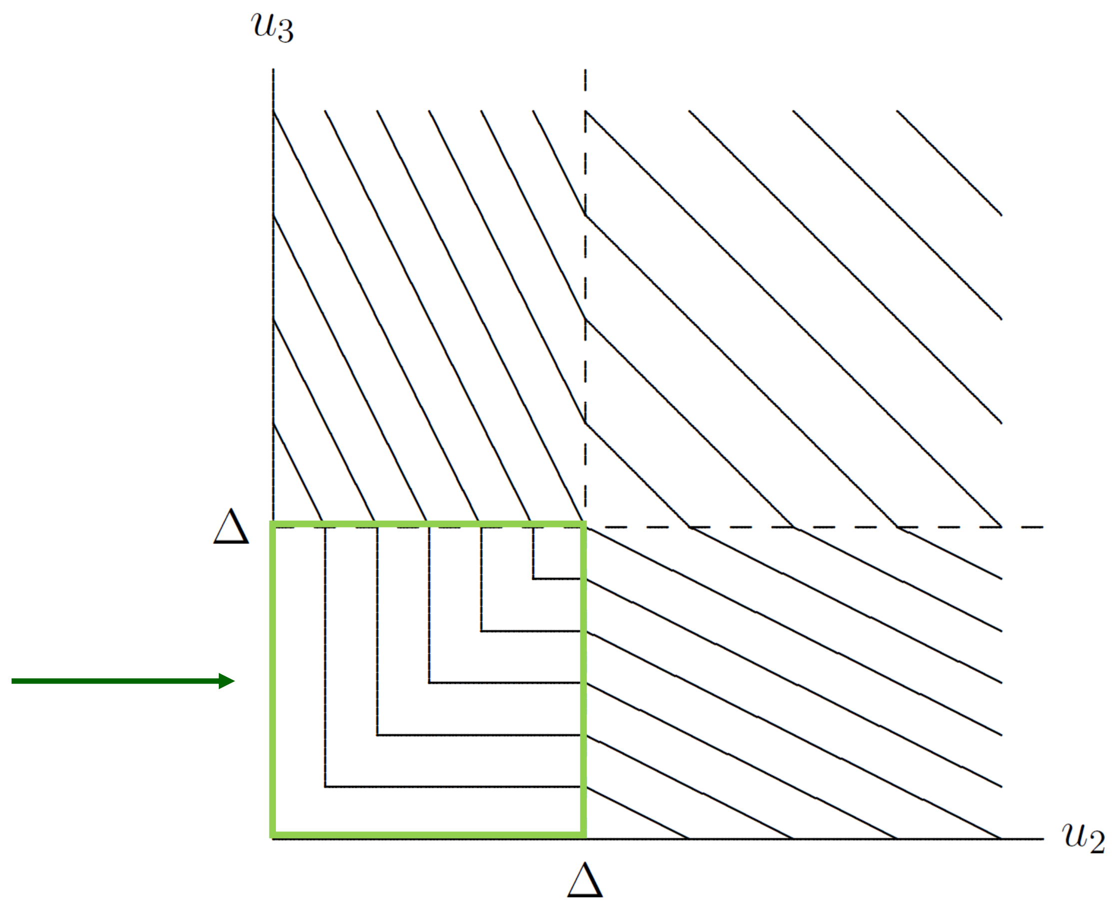
$$F_k(u) = (n-k+1)\min\left\{u_{\langle 1 \rangle} + \Delta,\ u_{\langle k \rangle}\right\} + \sum_{i=1}^{n}\left(u_{\langle i \rangle} - u_{\langle 1 \rangle} - \Delta\right)^{+}, \quad k \geq 2$$

H-W Model

Example: 3 persons

Contours for $F_2(0,u_2,u_3)$

Model is sensitive to equity of all persons in maximin region



Proposed Model

MILP model to maximize $F_k(u)$

$\bar{u}_{i_k} = u_i$ determined by maximizing $F_k(u)$

$I_k = \{1,\ldots,n\} \setminus \{i_1,\ldots,i_{k-1}\}$

$$\max \ z$$

$$z \le (n-k+1)\sigma + \sum_{i \in I_k} v_i$$

$$0 \le v_i \le M\delta_i, \ \ i \in I_k$$

$$v_i \le u_i - \bar{u}_{i_1} - \Delta + M(1-\delta_i), \ \ i \in I_k$$

$$\sigma \le u_{i_1} + \Delta$$

$$\sigma \le w$$

$$w \le u_i, \ \ i \in I_k$$

$$u_i \le w + M(1-\varepsilon_i), \ \ i \in I_k$$

$$\sum_{i \in I_k} \varepsilon_i = 1$$

$$w \ge \bar{u}_{i_{k-1}}$$

$$u_i - \bar{u}_{i_{k-1}} \le M, \ \ i \in I_k$$

$$\delta_i, \varepsilon_i \in \{0,1\}$$

Proposed Model

Theorem. The MILP model is correct.

The model is not sharp, but there are valid inequalities:

$$z \leq (n-k+1)u_i + \beta \sum_{j \in I_k \setminus \{i\}} \left(u_j - u_{i_{k-1}}\right), \ \ i \in I_k$$

where $\beta = \left(1 - \frac{\Delta}{M}\right)\left(1 - \frac{\bar{u}_{i_{k-1}} - \bar{u}_{i_1}}{M}\right)^{-1}$

Health Example

Measure utility in **QALY**s (quality-adjusted life years).

QALY and cost data based on Briggs & Gray, (2000) etc.

Decide whether to **fund** each disease/treatment pair.

Distinguish **severity levels** of each disease.

Treatment decisions are discrete, so funding is **all-or-nothing** for each category.

Health Example

Add constraints to define feasible set…

max z

model for $F_k(u)$

(modified for patient groups of different sizes)

$$\bar{u}_i = q_i y_i + \alpha_i$$

$$\sum_i n_i c_i y_i \le \text{budget}$$

$$y_i \in \{0,1\}, \ \text{all } i$$

y_i indicates whether subgroup i is funded

u_1

QALY & cost data

Part 1

Intervention	Cost per person c_i (£)	QALYs gained q_i	Cost per QALY (£)	QALYs without intervention α_i	Subgroup size n_i
Pacemaker for atrioventricular heart block					
Subgroup A	3500	3	1167	13	35
Subgroup B	3500	5	700	10	45
Subgroup C	3500	10	350	5	35
Hip replacement					
Subgroup A	3000	2	1500	3	45
Subgroup B	3000	4	750	4	45
Subgroup C	3000	8	375	5	45
Valve replacement for aortic stenosis					
Subgroup A	4500	3	1500	2.5	20
Subgroup B	4500	5	900	3	20
Subgroup C	4500	10	450	3.5	20
CABG[1] for left main disease					
Mild angina	3000	1.25	2400	4.75	50
Moderate angina	3000	2.25	1333	3.75	55
Severe angina	3000	2.75	1091	3.25	60
CABG for triple vessel disease					
Mild angina	3000	0.5	6000	5.5	50
Moderate angina	3000	1.25	2400	4.75	55
Severe angina	3000	2.25	1333	3.75	60
CABG for double vessel disease					
Mild angina	3000	0.25	12,000	5.75	60
Moderate angina	3000	0.75	4000	5.25	65
Severe angina	3000	1.25	2400	4.75	70

QALY & cost data

Part 2

Intervention	Cost per person c_i (£)	QALYs gained q_i	Cost per QALY (£)	QALYs without intervention α_i	Subgroup size n_i
	22,500	4.5	5000	1.1	2
Kidney transplant					
Subgroup A	15,000	4	3750	1	8
Subgroup B	15,000	6	2500	1	8
Kidney dialysis					
Less than 1 year survival					
Subgroup A	5000	0.1	50,000	0.3	8
1-2 years survival					
Subgroup B	12,000	0.4	30,000	0.6	6
2-5 years survival					
Subgroup C	20,000	1.2	16,667	0.5	4
Subgroup D	28,000	1.7	16,471	0.7	4
Subgroup E	36,000	2.3	15,652	0.8	4
5-10 years survival					
Subgroup F	46,000	3.3	13,939	0.6	3
Subgroup G	56,000	3.9	14,359	0.8	2
Subgroup H	66,000	4.7	14,043	0.9	2
Subgroup I	77,000	5.4	14,259	1.1	2
At least 10 years survival					
Subgroup J	88,000	6.5	13,538	0.9	2
Subgroup K	100,000	7.4	13,514	1.0	1
Subgroup L	111,000	8.2	13,537	1.2	1

H-W Results

Total budget £3 million

Δ range	Pace-maker	Hip repl.	Aortic valve	CABG L	CABG 3	CABG 2	Heart trans.	Kidney trans.	Kidney dialysis <1	Kidney dialysis 1-2	Kidney dialysis 2-5	Kidney dialysis 5-10	Kidney dialysis >10	Avg. QALYs.
0-3.3	111	111	111	111	111	111	1	11	0	0	000	0000	000	7.54
3.4-4.0	111	111	111	111	111	111	0	11	1	0	000	0000	000	7.54
4.0-4.4	111	111	111	111	111	111	0	01	1	0	000	0000	001	7.51
4.5-5.01	111	011	111	111	111	111	1	01	1	0	000	0000	011	7.43
5.02-5.55	111	011	011	111	111	111	1	01	1	0	000	0001	011	7.36
5.56-5.58	111	011	011	111	111	011	0	01	1	0	000	0001	111	7.36
5.59	111	011	011	110	111	111	0	01	1	0	000	0001	111	7.20
5.60-13.1	111	111	111	101	000	000	1	11	1	0	111	1111	111	7.06
13.2-14.2	111	011	111	011	000	000	1	11	1	1	111	1111	111	7.03
14.3-15.4	111	111	111	011	000	000	1	11	1	1	101	1111	111	7.13
15.5 up	111	011	111	011	001	000	1	11	1	0	011	1111	111	7.19

H-W Results

Utilitarian solution

Δ range	Pace-maker	Hip repl.	Aortic valve	CABG L	CABG 3	CABG 2	Heart trans.	Kidney trans.	Kidney dialysis <1	Kidney dialysis 1-2	Kidney dialysis 2-5	Kidney dialysis 5-10	Kidney dialysis >10	Avg. QALYs.
0-3.3	111	111	111	111	111	111	1	11	0	0	000	0000	000	7.54
3.4-4.0	111	111	111	111	111	111	0	11	1	0	000	0000	000	7.54
4.0-4.4	111	111	111	111	111	111	0	01	1	0	000	0000	001	7.51
4.5-5.01	111	011	111	111	111	111	1	01	1	0	000	0000	011	7.43
5.02-5.55	111	011	011	111	111	111	1	01	1	0	000	0001	011	7.36
5.56-5.58	111	011	011	111	111	011	0	01	1	0	000	0001	111	7.36
5.59	111	011	011	110	111	111	0	01	1	0	000	0001	111	7.20
5.60-13.1	111	111	111	101	000	000	1	11	1	0	111	1111	111	7.06
13.2-14.2	111	011	111	011	000	000	1	11	1	1	111	1111	111	7.03
14.3-15.4	111	111	111	011	000	000	1	11	1	1	101	1111	111	7.13
15.5 up	111	011	111	011	001	000	1	11	1	0	011	1111	111	7.19

H-W Results

Maximin solution

Δ range	Pace-maker	Hip repl.	Aortic valve	CABG L	CABG 3	CABG 2	Heart trans.	Kidney trans.	Kidney dialysis <1	Kidney dialysis 1-2	Kidney dialysis 2-5	Kidney dialysis 5-10	Kidney dialysis >10	Avg. QALYs.
0-3.3	111	111	111	111	111	111	1	11	0	0	000	0000	000	7.54
3.4-4.0	111	111	111	111	111	111	0	11	1	0	000	0000	000	7.54
4.0-4.4	111	111	111	111	111	111	0	01	1	0	000	0000	001	7.51
4.5-5.01	111	011	111	111	111	111	1	01	1	0	000	0000	011	7.43
5.02-5.55	111	011	011	111	111	111	1	01	1	0	000	0001	011	7.36
5.56-5.58	111	011	011	111	111	011	0	01	1	0	000	0001	111	7.36
5.59	111	011	011	110	111	111	0	01	1	0	000	0001	111	7.20
5.60-13.1	111	111	111	101	000	000	1	11	1	0	111	1111	111	7.06
13.2-14.2	111	011	111	011	000	000	1	11	1	1	111	1111	111	7.03
14.3-15.4	111	111	111	011	000	000	1	11	1	1	101	1111	111	7.13
15.5 up	111	011	111	011	001	000	1	11	1	0	011	1111	111	7.19

H-W Results

More dialysis with larger Δ, beginning with longer life span

Δ range	Pace-maker	Hip repl.	Aortic valve	CABG L	CABG 3	CABG 2	Heart trans.	Kidney trans.	Kidney dialysis <1	1-2	2-5	5-10	>10	Avg. QALYs.
0-3.3	111	111	111	111	111	111	1	11	0	0	000	0000	000	7.54
3.4-4.0	111	111	111	111	111	111	0	11	1	0	000	0000	000	7.54
4.0-4.4	111	111	111	111	111	111	0	01	1	0	000	0000	001	7.51
4.5-5.01	111	011	111	111	111	111	1	01	1	0	000	0000	011	7.43
5.02-5.55	111	011	011	111	111	111	1	01	1	0	000	0001	011	7.36
5.56-5.58	111	011	011	111	111	011	0	01	1	0	000	0001	111	7.36
5.59	111	011	011	110	111	111	0	01	1	0	000	0001	111	7.20
5.60-13.1	111	111	111	101	000	000	1	11	1	0	111	1111	111	7.06
13.2-14.2	111	011	111	011	000	000	1	11	1	1	111	1111	111	7.03
14.3-15.4	111	111	111	011	000	000	1	11	1	1	101	1111	111	7.13
15.5 up	111	011	111	011	001	000	1	11	1	0	011	1111	111	7.19

H-W Results

Maximin solutions largely utilitarian due to tie breaking

Δ range	Pace-maker	Hip repl.	Aortic valve	CABG L	CABG 3	CABG 2	Heart trans.	Kidney trans.	Kidney dialysis <1	Kidney dialysis 1-2	Kidney dialysis 2-5	Kidney dialysis 5-10	Kidney dialysis >10	Avg. QALYs.
0-3.3	111	111	111	111	111	111	1	11	0	0	000	0000	000	7.54
3.4-4.0	111	111	111	111	111	111	0	11	1	0	000	0000	000	7.54
4.0-4.4	111	111	111	111	111	111	0	01	1	0	000	0000	001	7.51
4.5-5.01	111	011	111	111	111	111	1	01	1	0	000	0000	011	7.43
5.02-5.55	111	011	011	111	111	111	1	01	1	0	000	0001	011	7.36
5.56-5.58	111	011	011	111	111	011	0	01	1	0	000	0001	111	7.36
5.59	111	011	011	110	111	111	0	01	1	0	000	0001	111	7.20
5.60-13.1	111	111	111	101	000	000	1	11	1	0	111	1111	111	7.06
13.2-14.2	111	011	111	011	000	000	1	11	1	1	111	1111	111	7.03
14.3-15.4	111	111	111	011	000	000	1	11	1	1	101	1111	111	7.13
15.5 up	111	011	111	011	001	000	1	11	1	0	011	1111	111	7.19

New Results

Pure leximax

Solution time < 0.5 sec for each *Δ*

Δ range	Pace-maker	Hip repl.	Aortic valve	CABG L	CABG 3	CABG 2	Heart trans.	Kidney trans.	Kidney dialysis <1	1-2	2-5	5-10	>10	Avg. QALYs.
0-3.24	111	111	111	111	111	111	1	11	0	0	000	0000	000	7.544
3.25-3.65	111	111	111	111	111	111	0	11	1	0	000	0000	000	7.535
3.66-4.41	111	111	111	111	111	111	1	01	1	0	100	0000	000	7.515
4.42-5.43	111	111	111	111	111	011	1	01	1	0	110	0000	000	7.506
5.44-5.50	111	111	111	111	011	001	0	01	1	0	110	0001	100	7.438
5.51-5.53	111	111	111	011	101	001	1	01	1	0	110	1011	100	7.348
5.54-5.56	111	111	111	011	001	010	1	01	1	1	111	0111	100	7.286
5.57-6.59	111	111	111	011	001	000	1	01	1	1	111	1111	100	7.243
6.60-7.07	111	111	111	011	101	000	1	00	1	1	111	1111	100	7.220
7.08-8.10	011	111	111	011	001	010	1	00	1	1	111	1111	100	7.128
8.11-8.61	011	111	111	011	001	001	1	00	1	1	111	1111	100	7.171
8.62-9.34	010	111	111	011	001	011	1	00	1	1	111	1111	100	6.838
9.35-11.21	000	111	111	011	101	011	1	00	1	1	111	1111	100	6.621
11.22-12.88	100	110	111	011	011	011	1	00	1	1	111	1111	100	6.392
12.89 up	000	110	111	111	111	001	1	00	1	1	111	1111	100	6.322

New Results

Genuine equity-utility tradeoff

Δ range	Pace-maker	Hip repl.	Aortic valve	CABG L	CABG 3	CABG 2	Heart trans.	Kidney trans.	Kidney dialysis <1	Kidney dialysis 1-2	Kidney dialysis 2-5	Kidney dialysis 5-10	Kidney dialysis >10	Avg. QALYs.
0-3.24	111	111	111	111	111	111	1	11	0	0	000	0000	000	7.544
3.25-3.65	111	111	111	111	111	111	0	11	1	0	000	0000	000	7.535
3.66-4.41	111	111	111	111	111	111	1	01	1	0	100	0000	000	7.515
4.42-5.43	111	111	111	111	111	011	1	01	1	0	110	0000	000	7.506
5.44-5.50	111	111	111	111	011	001	0	01	1	0	110	0001	100	7.438
5.51-5.53	111	111	111	011	101	001	1	01	1	0	110	1011	100	7.348
5.54-5.56	111	111	111	011	001	010	1	01	1	1	111	0111	100	7.286
5.57-6.59	111	111	111	011	001	000	1	01	1	1	111	1111	100	7.243
6.60-7.07	111	111	111	011	101	000	1	00	1	1	111	1111	100	7.220
7.08-8.10	011	111	111	011	001	010	1	00	1	1	111	1111	100	7.128
8.11-8.61	011	111	111	011	001	001	1	00	1	1	111	1111	100	7.171
8.62-9.34	010	111	111	011	001	011	1	00	1	1	111	1111	100	6.838
9.35-11.21	000	111	111	011	101	011	1	00	1	1	111	1111	100	6.621
11.22-12.88	100	110	111	011	011	011	1	00	1	1	111	1111	100	6.392
12.89 up	000	110	111	111	111	001	1	00	1	1	111	1111	100	6.322

Hardship cases come in earlier

New Results

Δ	Pace-	Hip	Aortic	CABG			Heart	Kidney	Kidney dialysis					Avg.
range	maker	repl.	valve	L	3	2	trans.	trans.	<1	1-2	2-5	5-10	>10	QALYs.
0-3.24	111	111	111	111	111	111	1	11	0	0	000	0000	000	7.544
3.25-3.65	111	111	111	111	111	111	0	11	1	0	000	0000	000	7.535
3.66-4.41	111	111	111	111	111	111	1	01	1	0	100	0000	000	7.515
4.42-5.43	111	111	111	111	111	011	1	01	1	0	110	0000	000	7.506
5.44-5.50	111	111	111	111	011	001	0	01	1	0	110	0001	100	7.438
5.51-5.53	111	111	111	011	101	001	1	01	1	0	110	1011	100	7.348
5.54-5.56	111	111	111	011	001	010	1	01	1	1	111	0111	100	7.286
5.57-6.59	111	111	111	011	001	000	1	01	1	1	111	1111	100	7.243
6.60-7.07	111	111	111	011	101	000	1	00	1	1	111	1111	100	7.220
7.08-8.10	011	111	111	011	001	010	1	00	1	1	111	1111	100	7.128
8.11-8.61	011	111	111	011	001	001	1	00	1	1	111	1111	100	7.171
8.62-9.34	010	111	111	011	001	011	1	00	1	1	111	1111	100	6.838
9.35-11.21	000	111	111	011	101	011	1	00	1	1	111	1111	100	6.621
11.22-12.88	100	110	111	011	011	011	1	00	1	1	111	1111	100	6.392
12.89 up	000	110	111	111	111	001	1	00	1	1	111	1111	100	6.322

Hardship cases stay in & displace pacemaker

New Results

Δ range	Pace-maker	Hip repl.	Aortic valve	CABG L	CABG 3	CABG 2	Heart trans.	Kidney trans.	Kidney dialysis <1	1-2	2-5	5-10	>10	Avg. QALYs.
0-3.24	111	111	111	111	111	111	1	11	0	0	000	0000	000	7.544
3.25-3.65	111	111	111	111	111	111	0	11	1	0	000	0000	000	7.535
3.66-4.41	111	111	111	111	111	111	1	01	1	0	100	0000	000	7.515
4.42-5.43	111	111	111	111	111	011	1	01	1	0	110	0000	000	7.506
5.44-5.50	111	111	111	111	011	001	0	01	1	0	110	0001	100	7.438
5.51-5.53	111	111	111	011	101	001	1	01	1	0	110	1011	100	7.348
5.54-5.56	111	111	111	011	001	010	1	01	1	1	111	0111	100	7.286
5.57-6.59	111	111	111	011	001	000	1	01	1	1	111	1111	100	7.243
6.60-7.07	111	111	111	011	101	000	1	00	1	1	111	1111	100	7.220
7.08-8.10	011	111	111	011	001	010	1	00	1	1	111	1111	100	7.128
8.11-8.61	011	111	111	011	001	001	1	00	1	1	111	1111	100	7.171
8.62-9.34	010	111	111	011	001	011	1	00	1	1	111	1111	100	6.838
9.35-11.21	000	111	111	011	101	011	1	00	1	1	111	1111	100	6.621
11.22-12.88	100	110	111	011	011	011	1	00	1	1	111	1111	100	6.392
12.89 up	000	110	111	111	111	001	1	00	1	1	111	1111	100	6.322

Shelter Location

Locate shelters for emergency (earthquake), and **assign** each neighborhood to a shelter.

Data from Mostajabdaveh, Gutjahr, and Salman (2019).

Remove stochastic elements.

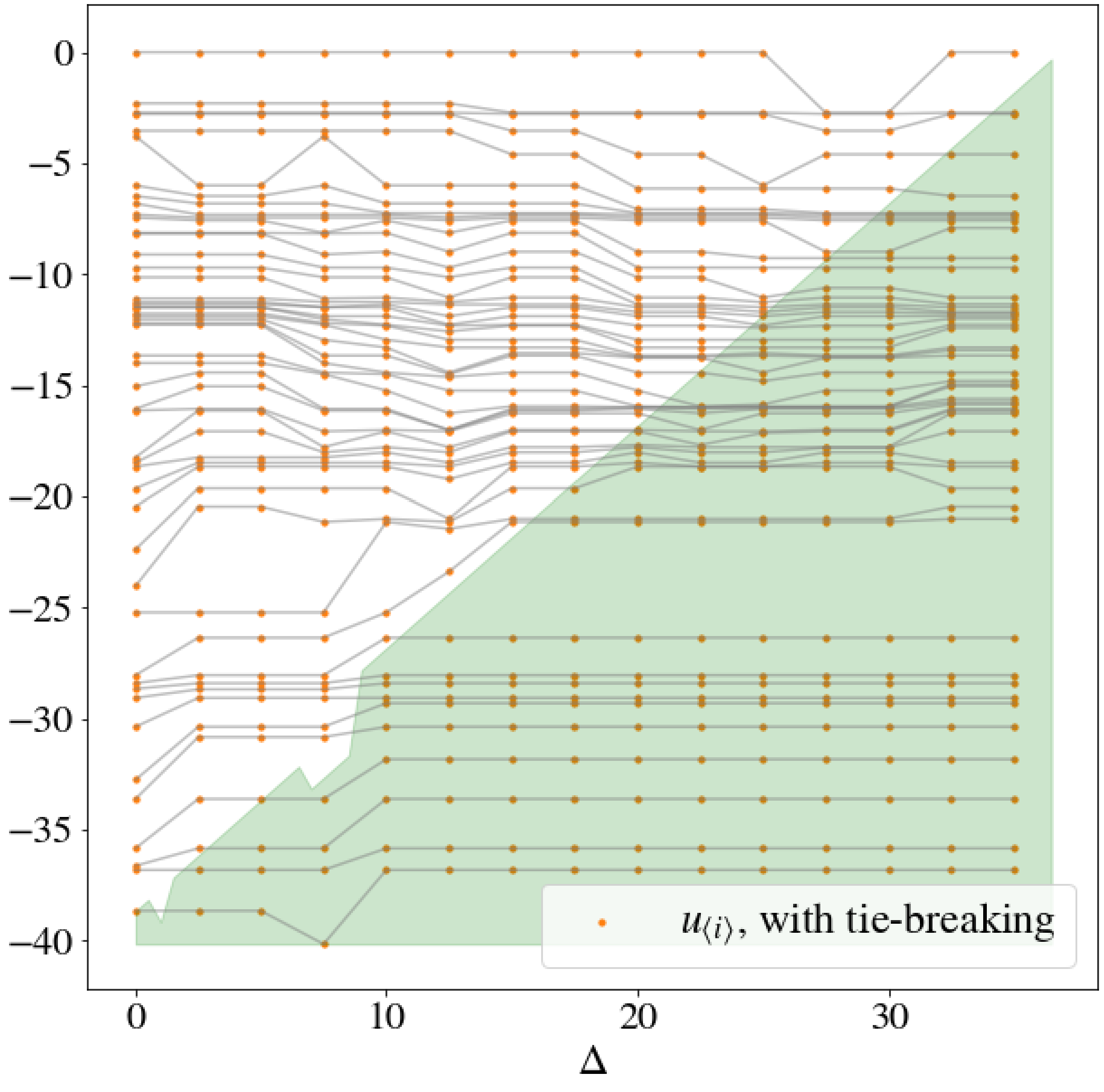
100 neighborhoods, 50 or 100 candidate shelter locations.

Some neighborhoods far from any candidate location.

Solution time < 10 sec for most *Δ*, always < 18 sec

100 neighborhoods

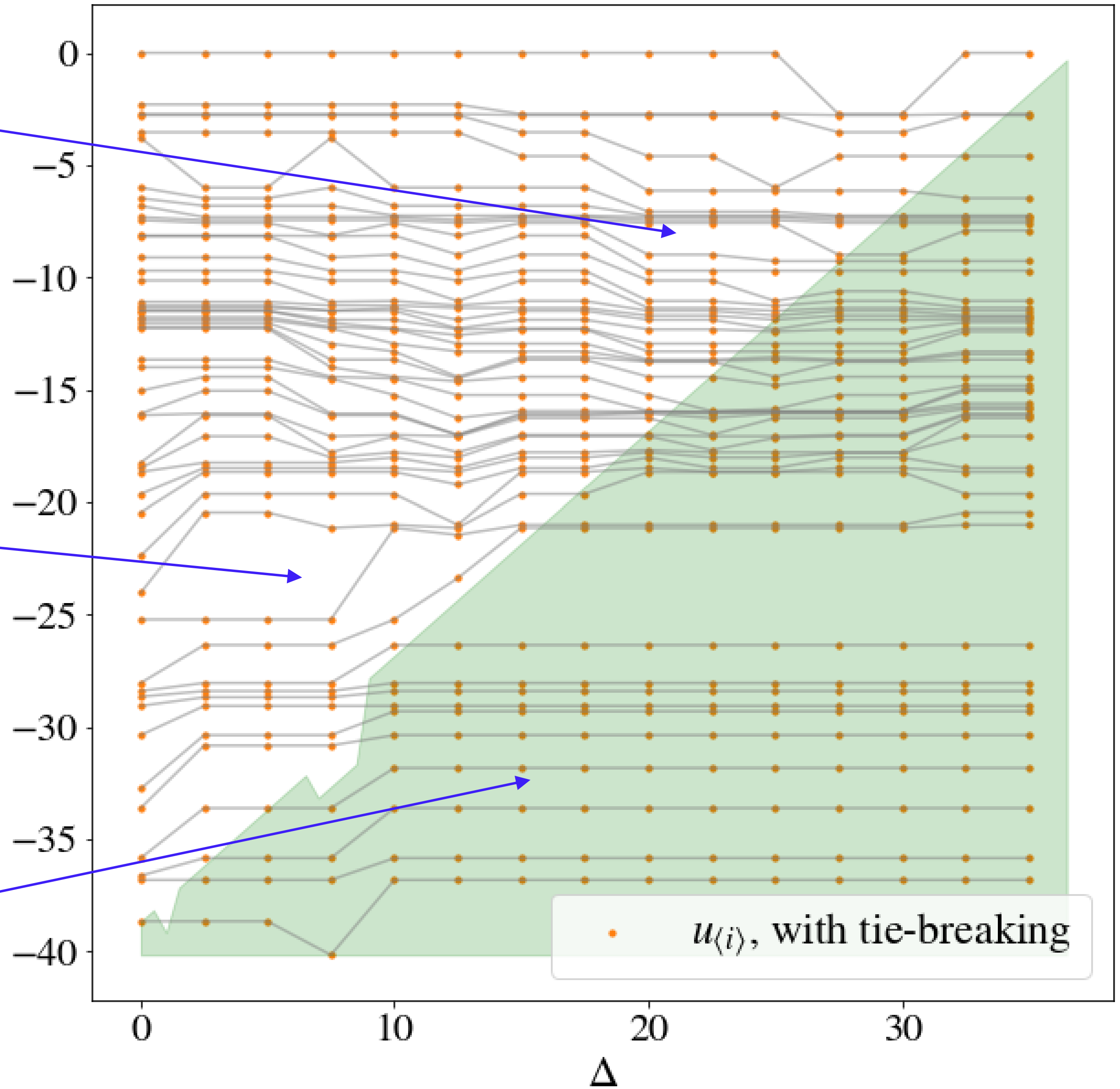
50 shelter locations



Easier-to-serve neighborhoods lose somewhat as *Δ* increases

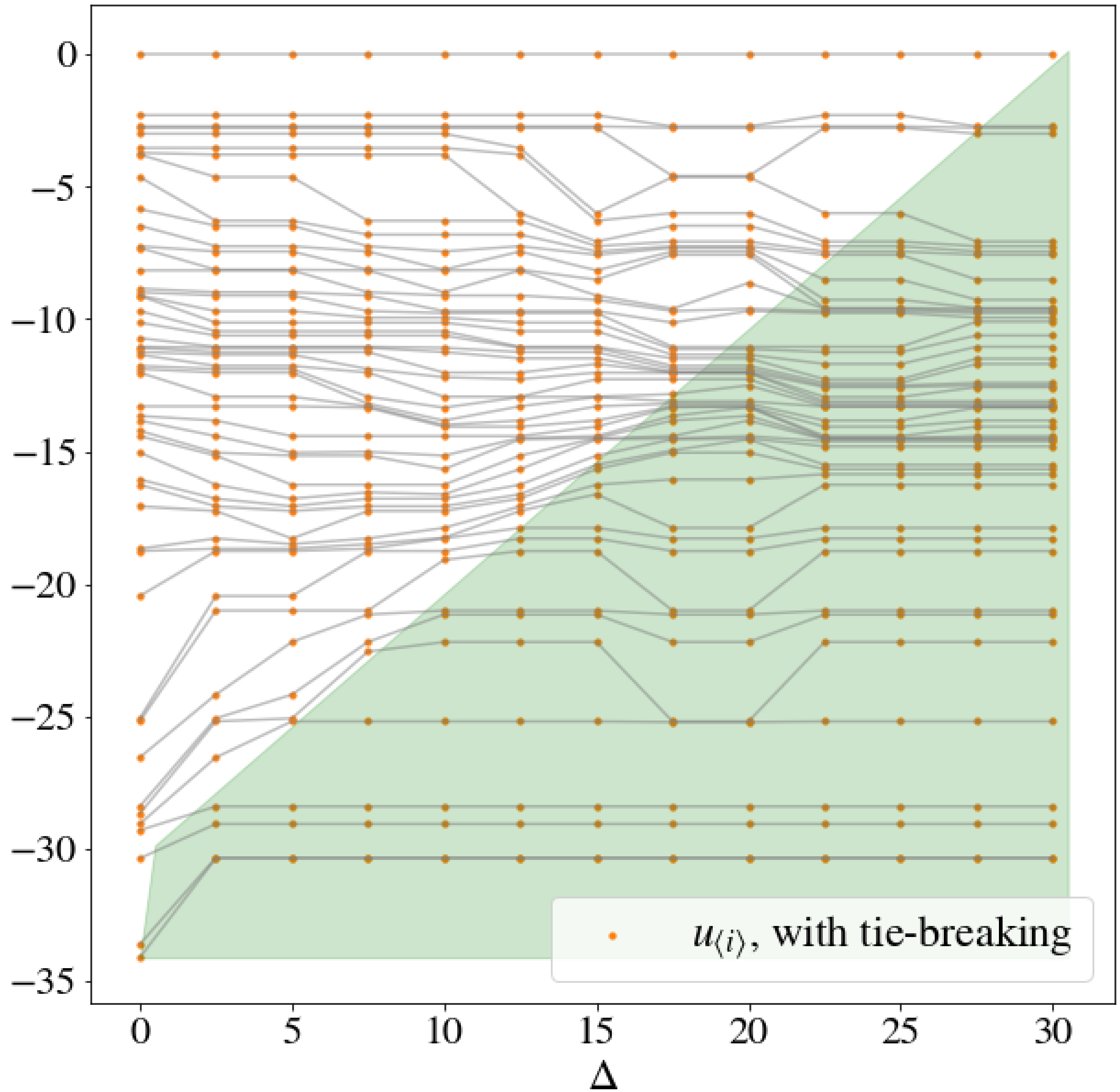
Additional disadvantaged nbhds receive priority as *Δ* increases

Some nbhds far from any shelter location. Model still prioritizes less disadvantaged neighborhoods.

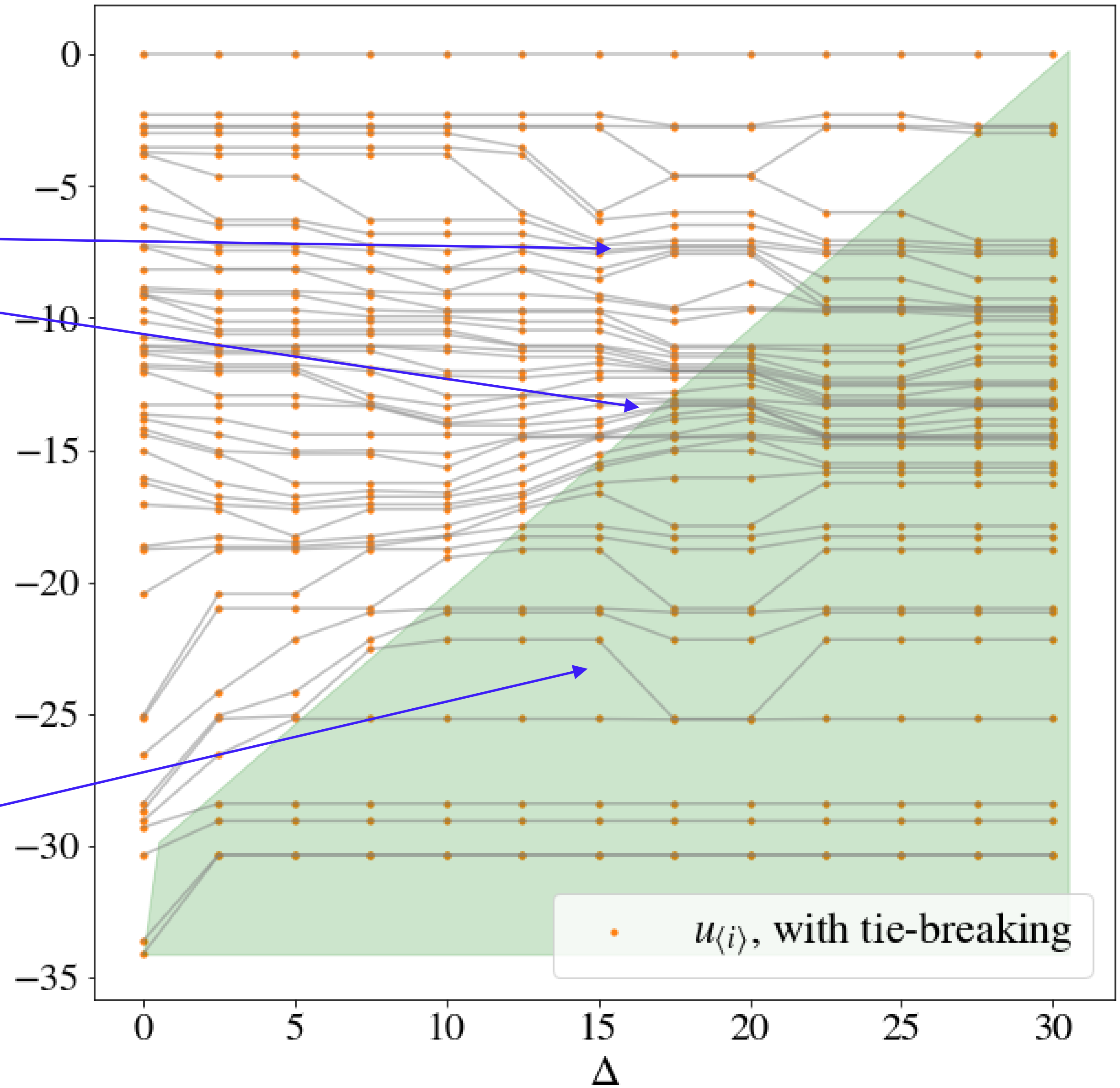


100 neighborhoods

100 shelter locations



Some relatively advantaged nbhds improve status temporarily as Δ increases, then lose as more nbhds enter fair region.



Some less advantaged nbhds lose temporarily as more nbhds enter fair region

Reference

V. Chen and J. N. Hooker, Balancing fairness and Efficiency in an optimization model, submitted (available on arXiv).