Last-Mile Scheduling Under Uncertainty

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Abstract. Shared mobility is revolutionizing urban transportation and has sparked interest in optimizing the joint schedule of passengers using public transit and last-mile services. Scheduling systems must anticipate future requests and provision flexibility in order to be adopted in practice. In this work, we consider a two-stage stochastic programming formulation for scheduling a set of known passengers and uncertain passengers that are realized from a finite set of scenarios. We present an optimization approach based on decision diagrams. We obtain, in minutes, schedules for 1,000 known passengers that are robust and optimized with respect to scenarios involving up to 100 additional uncertain passengers.

Keywords: Scheduling · Sample average · Decision diagrams

1 Introduction

The transportation industry is transforming due to recently introduced mechanisms for shared mobility [8, 14]. Transportation systems are a key element of integrative smart city operations, leading to a host of complex optimization problems [11, 17–19]. Of critical importance is the joint scheduling of mass transportation systems with last-mile vehicles, which when scheduled in unison can lead to significant operational improvements [13, 15, 16, 20].

This paper studies the joint scheduling of passengers on mass transit systems and last-mile vehicles under uncertainty. Passengers arrive by train to a central terminal and board limited-capacity pods called *commuter vehicles* (CVs) which are automated or otherwise operated, where some passengers are known and other passengers are uncertain (and thus may or may not request service). The goal is to assign passengers to trains and then to group passengers traveling together on a CV so as to minimize the convex combination of two objectives; total travel time and number of CV trips.[5] The mixed objective models a tradeoff

[5] We assume a single destination per CV trip [15, 16] and only a few destinations [13], which is operationally favorable since destination batching leads to efficiency.

between quality of service (total passenger travel time) and operational costs related to fuel consumption and maintenance requirements (number of CV trips). In the absence of uncertainty, this problem is known as the *integrated last-mile transportation problem* (ILMTP).

The uncertain setting is applicable to systems where a central scheduler takes requests from passengers and assigns them to trains, groups, and departure times, while building in flexibility for passengers that might request transportation services but are yet to submit an associated request. This leads to a significant increase in problem complexity with respect to previous work in this area [13, 15, 16], but also makes for a more realistic setting where the initial scheduling of passengers must account for additional demand from late requests that also needs to be accommodated.

Optimization under uncertainty, or *stochastic optimization*, defines more challenging mathematical problems [7]. A relatively recent and popular technique for handling uncertainty is robust optimization [2], where an uncertainty set is defined and worst-case operational decisions are employed. It is well known that this can lead to highly conservative solutions, since unlikely outcomes can drive decisions. A more classical approach consists of simulation-based optimization algorithms such as sample average approximation methods [10], which consider a finite set of possible realizations described as a sample of scenarios which are optimized over in order to maximize the expected value over the sampling.

This paper presents a two-stage stochastic programming formulation for the ILMTP under additional passenger uncertainty (ILMTP-APU). We assume that passenger requests consist of a set of *known* passengers and *uncertain* passengers. The uncertain passengers are modeled through a finite set of scenarios. The first-stage decision is the scheduling of known passengers and the second-stage schedules the additional passengers from a finite set of scenarios. Our approach relies on decision-diagram (DD)-based optimization (DDO) [1, 3, 6], and more specifically on decompositions based on DDs [4, 5], inspired by the model presented in [15]. Specifically, a DD is built for known passengers going to each building and separately for unknown passengers in each scenario and for each building. The DDs are then integrated through channeling constraints that can be optimized over through an integer programming (IP) formulation. This results in a large model. However, due to the tightness of the formulation, we obtain a reliable approach for optimally solving the problem. The resulting solutions are significantly better than what could be obtained by solving the problem for the known passengers to optimality and then scheduling the unknown passengers with the remaining capacity when a scenario is realized.

This paper adds to the recent literature on DDO for stochastic optimization, where BDDs have been used for determining decision-dependnent scenario probabilities [9] and, more closely related to the current study, a study of a class of two-stage stochastic IP problems [12]. Our proposed approach differs from that of [12] in that we model both the first-state and second-stage decisions using DDs and link them through assignment constraints. This provides an additional mechanism by which decision making uncertainty can be tackled through DDs. The main contributions of this paper are therefore (1) an extension of the ILMTP to incorporate uncertain passenger arrivals, (2) structural results on families of optimal solutions and (3) the development of a novel DDO stochastic programming modeling framework for solving the problem based on such structural results. An experimental evaluation on synthetic instances shows great promise. In particular, the solutions obtained are far superior to a basic heuristic extension of the work in [15] and the algorithm scales to 100 uncertain passengers per scenario when 1,000 confirmed passengers are scheduled.

2 Problem Description

We first describe the elements of the problem, including the mass transit system, last-mile vehicles, destinations, passenger requests, and associated parameters.
Mass transit system: For the sake of convenience, we will assume that the mass transit is a train system. Let $\mathbf{T}_0$ be the *terminal station* that links a mass transit system with a last-mile service system. The mass transit system is described by a set of *trips*, denoted by $\mathcal{C}$, with $n_c := |\mathcal{C}|$. Each trip originates at a station in set $\mathcal{S}$ and ends at $\mathbf{T}_0$. The trips are regular in the sense that the train stops at all stations in $\mathcal{S}$ sequentially, with $\mathbf{T}_0$ as the last stop of each trip. A trip c leaves station $s \in \mathcal{S}$ at time $\tilde{t}(c,s)$ and arrives to $\mathbf{T}_0$ at $\tilde{t}(c)$.
Destinations: Let $\mathcal{D}$ be the set of destinations where the CVs make stops, with $K := |\mathcal{D}|$, where we assume $\mathbf{T}_0 \in \mathcal{D}$. For each destination $d \in \mathcal{D}$, let $\tau(d)$ be the total time it takes a CV to depart $\mathbf{T}_0$, travel to d (denoted by $\tau^1(d)$), stop at d for passengers to disembark (denoted by $\tau^2(d)$), and return to $\mathbf{T}_0$ (denoted by $\tau^3(d)$). Therefore, $\tau(d) = \tau^1(d) + \tau^2(d) + \tau^3(d)$. Let $\mathcal{T} := \{1, \ldots, t^{\max}\}$ be an index set of the operation times of both systems. We assume that the time required to board passengers into the CVs is incorporated in $\tau^1(d)$. For simplicity, the boarding time is independent of the number of passengers.
Last-mile system: Let V be the set of CVs, with $m := |V|$. Denote by v^{cap} the number of passengers that can be assigned to a single CV trip. Each CV trip consists of a set of passengers boarding the CV, traveling from $\mathbf{T}_0$ to a destination $d \in \mathcal{D}$, and then returning back to $\mathbf{T}_0$. Therefore, passengers sharing a common CV trip must request transportation to a common building. We also assume that each CV must be back at the terminal by time $t^{\max}$.
Known Passengers: Let $\mathcal{J}$ be the set of known passengers. Each passenger $j \in \mathcal{J}$ requests transport from a station $s(j) \in \mathcal{S}$ to $\mathbf{T}_0$, and then by CV to destination $d(j) \in \mathcal{D}$, to arrive at time $t^r(j)$. The set of passengers that request service to destination d is denoted by $\mathcal{J}(d)$. Let $n := |\mathcal{J}|$ and $n_d := |\mathcal{J}(d)|$. Each passenger $j \in \mathcal{J}$ must arrive to $d(j)$ between $t^r(j) - T_w$ and $t^r(j) + T_w$.
Uncertain Passengers: We assume a finite set of scenarios, denoted by $\mathcal{Q}$, representing different realizations of the uncertain passengers. Let $\widehat{\mathcal{J}}(q)$ be the set of uncertain passengers in scenario $q \in \mathcal{Q}$. Each passenger $j \in \widehat{\mathcal{J}}(q)$ requests transport from a station $s(j) \in \mathcal{S}$ to $\mathbf{T}_0$, and then by CV to destination $d(j) \in \mathcal{D}$, to arrive at time $t^r(j)$. The set of passengers that request service to destination d is denoted by $\widehat{\mathcal{J}}(q,d)$. Let $\widehat{n}_q := |\widehat{\mathcal{J}}(q)|$ and $\widehat{n}_{q,d} := \left|\widehat{\mathcal{J}}(q,d)\right|$.

Problem Statement: The ILMTP-APU is the problem of assigning train trips and CVs to each known passenger so that the uncertain passengers in any of the scenarios $\mathcal{Q}$ can be feasibly scheduled and the expected value of a convex combination of the total transit time and the number of CV trips utilized is minimized. A solution therefore consists of:

- a partition $\mathtt{g} = \{g_1, \ldots, g_\gamma\}$ of $\mathcal{J}$, with each group g_l associated with a departure time $t_l^{\mathtt{g}}$, for $l = 1, \ldots, \gamma$, indicating the time the CV carrying the passengers in g_l departs $\mathbf{T}_0$, satisfying all request time and operational constraints. For any passenger $j \in \mathcal{J}$, let $\mathtt{g}(j)$ be the group in $\mathtt{g}$ that j belongs to.
- for each $q \in \mathcal{Q}$, a partition $\widehat{\mathtt{g}}(q) = \left\{\widehat{g}_{q,1}, \ldots, \widehat{g}_{q,\widehat{\gamma}(q)}\right\}$ of $\widehat{\mathcal{J}}(q)$, with each group $\widehat{g}_{q,k}$ associated with a departure time $t_k^{\widehat{\mathtt{g}}(q)}$, for $k = 1, \ldots, \widehat{\gamma}(q)$ and an indicator function $\sigma(q,k) \in \{1, \ldots, \gamma\} \cup \{\emptyset\}$. $\sigma(q,k) \neq \emptyset$ indicates that uncertain passenger group $\widehat{g}_{q,k}$ shares the last-mile trip with known passenger group $g_{\sigma(q,k)}$. In other words, groups leave from the terminal at the same time (i.e. $t^{\mathtt{g}}_{\sigma(q,k)} = t_k^{\widehat{\mathtt{g}}(q)}$) and the combination of confirmed passenger group and all such shared passenger groups in a scenario does not exceed the CV capacity, i.e. $|g_l| + \sum_{k \in \{1,\ldots,\widehat{\gamma}(q) : \sigma(q,k) = l\}} |\widehat{g}_{q,k}| \leq v^{\text{cap}}$ for each $l \in \{1, \ldots, \gamma\}$ and $q \in \mathcal{Q}$.

3 Structure of Optimal Solutions

The deterministic version of ILMTP has optimal solutions with a structure that is particularly convenient to define more compact models. For each destination, passengers can be sorted by their desired arrival times and then grouped sequentially. This structure is valid to minimize passenger average waiting time [16] as well as the number of CV trips and the case in which travel times are time-dependent [15], hence leading to the compact DD-based model in [15].

In the case of ILMTP-APU, however, a more elaborate structure is required in order to represent optimal solutions. For example, let us suppose that for a particular time range there is a single CV of capacity 4 available, 4 known passengers, and just 1 unknown passenger in 1 out of 10 scenarios. Furthermore, let us assume that a first trip with the CV incurs no wait time whereas a second trip would impose a wait time of w on any passenger involved, and that the desired arrival time of the uncertain passenger falls strictly in the middle of those of the known passengers. If we sort and group all passengers regardless of their categories, then at least 2 trips will always be necessary and at least 1 known passenger has to wait w. But if we define a second trip only for the unknown passenger, then the average cost of the solution is reduced to a tenth because the second trip and the corresponding wait time w only materialize in 1 out of 10 scenarios. Since uncertain passengers have a smaller impact on the objective function, it becomes intuitive that they might be delayed with respect to known passengers as long as the schedule remains feasible for all passengers. We formalize this two-tier structure using the groups from the previous section.

Proposition 1. *When ILMTP-APU is feasible, there is an optimal solution where the groups of passengers for each category – known or uncertain from a scenario q – are grouped sequentially by their desired arrival times.*

Proof. Let us assume, by contradiction, that there is an instance for which the statement does not hold for a group of known passengers involving a passenger j with destination d in group g_1 and another group $g_2 \supseteq \{j-1, j_+\}$ for some $j_+ > j$. Furthermore, among the optimal solutions for such an instance, let us choose the optimal solution for which the first index d of the destination where such a grouping of known passengers does not exist is maximized; and among those solutions the one that maximizes the index j of the first passenger for which there is a group defined by passengers before and after j is maximized.

The key to obtaining a contradiction is the fact that the length of the time window for arrival is identical for all passengers each of whom have access to the same public transit service. Let us denote by $g_A \in \{g_1, g_2\}$ the group with earliest arrival time, say t_A; let us denote the other group by g_B, for which the departure time t_B is such that $t_B \geq t_A$; and let us denote the indices of passengers on either group as $\{j_1, j_2, \ldots, j_{|g_A|+|g_B|}\}$, where $t^r_{j_i} \leq t^r_{j_{i+1}}$ for $i = 1, \ldots, |g_A| + |g_B| - 1$. Note that all of these passengers have either t_A or t_B in their time windows. Since at least $|g_A|$ passengers can arrive at time t_A, it follows that the time window of the first $|g_A|$ passengers includes t_A. If not, then some of these first passengers would contain t_B in their time window instead, implying that some among the remaining $|g_B|$ passengers have t_A in their time window, and thus that the passengers are not sorted by desired arrival times. Thus, $t_A \in [t^r_{j_{|g_A|}} - \omega, t^r_{j_1} + \omega]$. Similarly, at least $|g_B|$ passengers can arrive at time t_B, and thus $t_B \in [t^r_{j_{|g_A|+|g_B|}} - \omega, t^r_{j_{|g_A|+1}} + \omega]$. Hence, we can replace the passengers of group g_A with $\{j_1, \ldots, j_{|g_A|}\}$ and those of group g_B with $\{j_{|g_A|+1}, \ldots, j_{|g_A|+|g_B|}\}$ while preserving their arrival times, size, and consequently with no change to the feasibility or optimality of the solution. However, this exhange implies that up to destination d and passenger j all passengers are grouped by sorted arrival times, hence contradicting the choice of d and j.

Without loss of generality, we can apply the same argument for uncertain passengers by also choosing the maximum index of a scenario q for which the groupings are not continuous and finding the same contradiction. □

The following result, which is independent from Proposition 1, is also helpful to simplify the modeling of ILMTP-APU.

Proposition 2. *When ILMTP-APU is feasible, there is an optimal solution where at most one group of uncertain passengers for each scenario is assigned to each each group of known passengers.*

Proof. If multiple groups are assigned, they have the same arrival times in any optimal solution and thus can be combined without loss of generality. □

4 Decision Diagram for Single Building

Following similar notation as in [15], we construct a decision diagram for each category of passenger and destination. For every $d \in \mathcal{D}$, we define a decision diagram $\mathsf{D}^d = (\mathsf{N}^d, \mathsf{A}^d)$ for the known passengers. N^d is partitioned into $n_d + 1$ ordered layers $\mathsf{L}_1^d, \mathsf{L}_2^d, \ldots, \mathsf{L}_{n_d+1}^d$ where $n_d = |\mathcal{J}(d)|$. Layer $\mathsf{L}_1^d = \{\mathbf{r}^d\}$ and layer $\mathsf{L}_{n_d+1}^d = \{\mathbf{t}^d\}$ consist of one node each; the *root* and *terminal*, respectively. The *layer* of node $\mathsf{u} \in \mathsf{L}_i^d$ is defined as $\ell(\mathsf{u}) = i$. Each arc $a \in \mathsf{A}^d$ is directed from its *arc-root* $\psi(a)$ to its *arc-terminal* $\omega(a)$, with $\ell(\psi(a)) = \ell(\omega(a)) - 1$. We denote the *arc-layer* of a as $\ell(a) := \ell(\psi(a))$. It is assumed that every maximal arc-directed path connects $\mathbf{r}^d$ to $\mathbf{t}^d$. Similarly, for each scenario $q \in \mathcal{Q}$, we define a decision diagram $\mathsf{D}^{d,q} = (\mathsf{N}^{d,q}, \mathsf{A}^{d,q})$ by using the corresponding upper index q for disambiguation.

The arcs between layers of the diagram correspond to the passengers that request transportation to the destination. Each node u is associated with a *state* $\mathsf{s}(\mathsf{u})$ corresponding to the number of immediately preceding passengers that is grouped with the next passenger. Each arc a is associated with a label $\phi(a) \in \{0,1\}$ on whether passenger $\psi(a)$ is not the last one in the group, in which case $\phi(a) = 0$ and $\mathsf{s}(\omega(a)) = \mathsf{s}(\psi(a)) + 1 \leq v^{\text{cap}}$, or else $\phi(a) = 1$. There can be multiple arcs between the same nodes in the latter case, each arc a corresponding to a different CV start time $t^0(a)$. Accordingly, each arc a such that $\phi(a) = 1$ has a corresponding wait time $W(a)$ for all passengers in the group and a label $\chi(a,t) \in \{0,1\}$ to indicate that a CV would be active at time t (i.e. $t^0(a) \leq t \leq t^0(a) + \tau(d)$) if arc a is chosen. Hence, $\phi(a) = 1$ implies $\chi(a,t) = 0$.

5 IP Formulation

We introduce binary variable $x_a \in \{0,1\} \, \forall \, a \in \mathsf{A}^d, d \in \mathcal{D}$, to denote the choice of the particular arc in the DD for known passengers. Similarly, we introduce binary variable $y_a^q \in \{0,1\} \, \forall \, a \in \mathsf{A}^{d,q}, d \in \mathcal{D}$ to denote the choice of the particular arc in the DD for uncertain passengers in scenario $q \in \mathcal{Q}$. Let $\widetilde{\mathsf{A}}^{d,q} = \{(a_1, a_2) \in \mathsf{A}^d \times \mathsf{A}^{d,q} \mid \phi(a_1) = 1, \phi(a_2) = 1, t^0(a_1) = t^0(a_2) \text{ and } \mathsf{s}(\psi(a_1)) + \mathsf{s}(\psi(a_2)) + 2 \leq v^{\text{cap}}\}$. The set $\widetilde{\mathsf{A}}^{d,q}$ denotes the set of feasible pairs of known passenger group and uncertain passenger group of scenario q, i.e. identical start time on the CV and the capacity constraint is satisfied. Let $z_{a_1,a_2}^q \in \{0,1\} \, \forall \, (a_1, a_2) \in \mathsf{A}^{d,q}$ denote the decision of pairing the group of known passengers represented by arc a_1 and group of uncertain passengers represented by arc a_2.

The objective function can be expressed as

$$f(\alpha) = \sum_{d \in \mathcal{D}} \sum_{a \in \mathsf{A}^d} [\alpha W(a) + (1-\alpha)] x_a + \frac{1}{|\mathcal{Q}|} \sum_{d \in \mathcal{D}} \sum_{q \in \mathcal{Q}} \left(\sum_{a \in \mathsf{A}^{d,q}} [\alpha W(a) + (1-\alpha)] y_a^q - (1-\alpha) \sum_{(a_1,a_2) \in \widetilde{\mathsf{A}}^{d,q}} z_{a_1,a_2}^q \right)$$

The following constraint imposes that only one group of uncertain passengers is paired with a group of known passengers if the latter is selected:

$$\sum_{a_2:(a_1,a_2)\in\widetilde{\mathsf{A}}^{d,q}} z^q_{a_1,a_2} \leq y_{a_1} \forall d\in\mathcal{D}, q\in\mathcal{Q}, a_1\in\mathsf{A}^d : \phi(a_1)=1. \tag{1a}$$

The fleet size constraint can be modeled for all $t\in\mathcal{T}, q\in\mathcal{Q}$ as

$$\sum_{d\in\mathcal{D}}\left(\sum_{a\in\mathsf{A}^d:\chi(a,t)=1} x_a + \sum_{a\in\mathsf{A}^{d,q}:\chi(a,t)=1} y^q_a\right) - \sum_{d\in\mathcal{D}}\sum_{(a_1,a_2)\in\widetilde{\mathsf{A}}^{dq}} z^q_{a_1,a_2} \leq m. \tag{1b}$$

The IP model for the ILMTP-APU is

$$\begin{aligned}
\min\quad & f(\alpha) & & \\
\text{s.t.}\quad & \text{Network flow constraints for } \mathsf{D}^d & \forall d\in\mathcal{D} & \quad (2a)\\
& \text{Network flow constraints for } \mathsf{D}^{d,q} & \forall d\in\mathcal{D}, q\in\mathcal{Q} & \quad (2b)\\
& \text{Eq. (1a), (1b)} & & \\
& x_a\in\{0,1\} & \forall d\in\mathcal{D}, a\in\mathsf{A}^d & \quad (2c)\\
& y^q_a\in\{0,1\} & \forall d\in\mathcal{D}, q\in\mathcal{Q}, a\in\mathsf{A}^d & \quad (2d)\\
& z^q_{a_1,a_2}\in\{0,1\} & \forall d\in\mathcal{D}, q\in\mathcal{Q}, (a_1,a_2)\in\widetilde{\mathsf{A}}^{d,q}. & \quad (2e)
\end{aligned}$$

The network flow constraints in (2a)-(2b) guarantee that a path is taken on each decision diagram, which corresponds to the groupings of known passengers and uncertain passengers for each scenario.

6 Experimental Results

This section provides an experimental evaluation of the algorithms developed in this paper. All experiments were run on a machine with an Intel(R) Core(TM) i7-4770 CPU @ 3.40GHz and 16 GB RAM. All algorithms were implemented in `Python 2.7.6` and the ILPs are solved using `Gurobi 7.5.1`. Simiar instances to those in [15] are generated, each with 1000 passengers, 60 CVs with capacity of 5 each, 10 destinations, and 10 scenarios each containing 50 or 100 passengers.

To provide a benchmark for which to compare the solutions obtained by our model, we suggest the following heuristic `H`: (1) solve the problem optimally for known passengers using the algorithm in [15]; and (2) for each scenario, solve the resulting MIP formulation to maximize the number of uncertain passengers that can be scheduled on average by using the remaining capacity on the trips already scheduled and the remaining availability of the CVs for additional trips.

Table 1 summarizes the experiments for $\alpha = 0$. Each instance $\mathrm{P}_{k,u,i}$ corresponds to the i-th instance with k known passengers and u unknown passengers on each of the 10 scenarios. We report the values for the first stage (known passengers) and for the second stage (uncertain passengers) as well as runtimes. If the second stage is infeasible, we report the percentage of scheduled passengers.

Table 1. Comparison of solution obtained with DDO and with heuristic H.

Instance	Heuristic H			DDO Approach		
	1st Stage	2nd Stage	Runtime	1st Stage	2nd Stage	Runtime
$P_{1000,50,1}$	204.0	(71.2%)	20.4	204.0	27.8	303.2
$P_{1000,50,2}$	204.0	(82.4%)	18.6	204.0	26.1	304.8
$P_{1000,50,3}$	204.0	(84.8%)	15.8	204.0	27.1	302.2
$P_{1000,50,4}$	205.0	(94.6%)	18.0	205.0	28.2	293.3
$P_{1000,50,5}$	205.0	(48.8%)	23.3	205.0	27.2	293.1
$P_{1000,50,6}$	203.0	(56.2%)	23.0	203.0	27.7	360.0
$P_{1000,50,7}$	203.0	(50.6%)	22.8	203.0	27.0	334.1
$P_{1000,50,8}$	204.0	(74.4%)	22.7	204.0	28.4	329.6
$P_{1000,50,9}$	204.0	(68.4%)	23.3	204.0	27.9	346.5
$P_{1000,50,10}$	203.0	(59.2%)	22.3	203.0	26.7	325.6
$P_{1000,100,1}$	203.0	(64.7%)	18.5	203.0	34.2	409.8
$P_{1000,100,2}$	205.0	(61.9%)	19.1	205.0	36.3	404.2
$P_{1000,100,3}$	204.0	(73.1%)	20.0	204.0	36.2	398.1
$P_{1000,100,4}$	204.0	(84.0%)	18.7	204.0	35.4	420.3
$P_{1000,100,5}$	204.0	(67.2%)	20.4	204.0	33.4	433.3
$P_{1000,100,6}$	205.0	(88.7%)	17.2	205.0	35.2	406.1
$P_{1000,100,7}$	203.0	(67.9%)	19.4	203.0	36.1	404.2
$P_{1000,100,8}$	204.0	(92.5%)	17.0	204.0	35.5	408.7
$P_{1000,100,9}$	203.0	(73.3%)	17.8	203.0	34.5	424.9
$P_{1000,100,10}$	203.0	(79.5%)	20.3	203.0	35.2	428.6

Note that the optimal solution of the deterministic case, which is used in the first stage of heuristic H, has the same value as the first stage of the DDO approach. Hence, there is no loss for the known passengers in the instances that we tested. In fact, the DDO approach found a solution that is robust for the scenarios considered while also optimal for the known passengers. Furthermore, ignoring the second stage leads to infeasible second stage problems in all cases for heuristic H, where only a portion of the uncertain passengers could be scheduled. Interestingly, the proportion of uncertain passengers that are scheduled when the optimal solution of the known passengers alone is fixed remains approximately the same for 50 and 100 uncertain passengers per scenario. Hence, reducing the number of uncertain passengers does not make heuristic H more suitable.

7 Conclusion

We considered a two-stage optimization problem of last-mile passenger scheduling subject to a finite set of scenarios representing uncertain additional demand. Our approach based on decision diagram optimization produces solutions that, despite an increase in runtimes, are feasibly robust with respect to all scenarios while minimizing the expected number of last-mile trips necessary to satisfy the demand across all scenarios. The results show the potential of using decision diagrams to solve such challenging problems of scheduling under uncertainty.

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