

# Combining Equity and Efficiency in Health Care

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# Equity and Efficiency

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**The problem is to find a fair and reasonable distribution of resources.**
- **Immediate motivation: Growing expense of health care interventions.**

# Equity and Efficiency

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**The problem is to find a fair and reasonable distribution of resources.**
- **Immediate motivation: Growing expense of health care interventions.**
- **Two classical criteria for distributive justice:**
  - **Utilitarianism (efficiency)**
  - **Difference principle of John Rawls (equity)**



# Equity and Efficiency

- © The McGraw-Hill Companies, Inc. All rights reserved. **Utilitarianism** seeks allocation of wealth to individuals that maximizes total utility.
  - ...and forbid very expensive treatments for seriously ill?

# Equity and Efficiency

- © The McGraw-Hill Companies, Inc. All rights reserved. **Utilitarianism** seeks allocation of wealth to individuals that maximizes total utility.
  - ...and forbid very expensive treatments for seriously ill?
- **The Rawlsian difference principle** seeks to maximize the welfare of the least advantaged.
  - ...at what cost to others?

# Combining Equity and Efficiency

- Utilitarian and Rawlsian distributions seem **too extreme** in practice.
  - How to combine them?

# Combining Equity and Efficiency

- Utilitarian and Rawlsian distributions seem **too extreme** in practice.
  - How to combine them?
- **One proposal:**
  - Maximize welfare of **most seriously ill** (Rawlsian)...
  - ...until this requires **undue sacrifice** from others

# Combining Equity and Efficiency

- In particular:
  - Switch from **Rawlsian** to **utilitarian** when **inequality** exceeds  $\Delta$ .

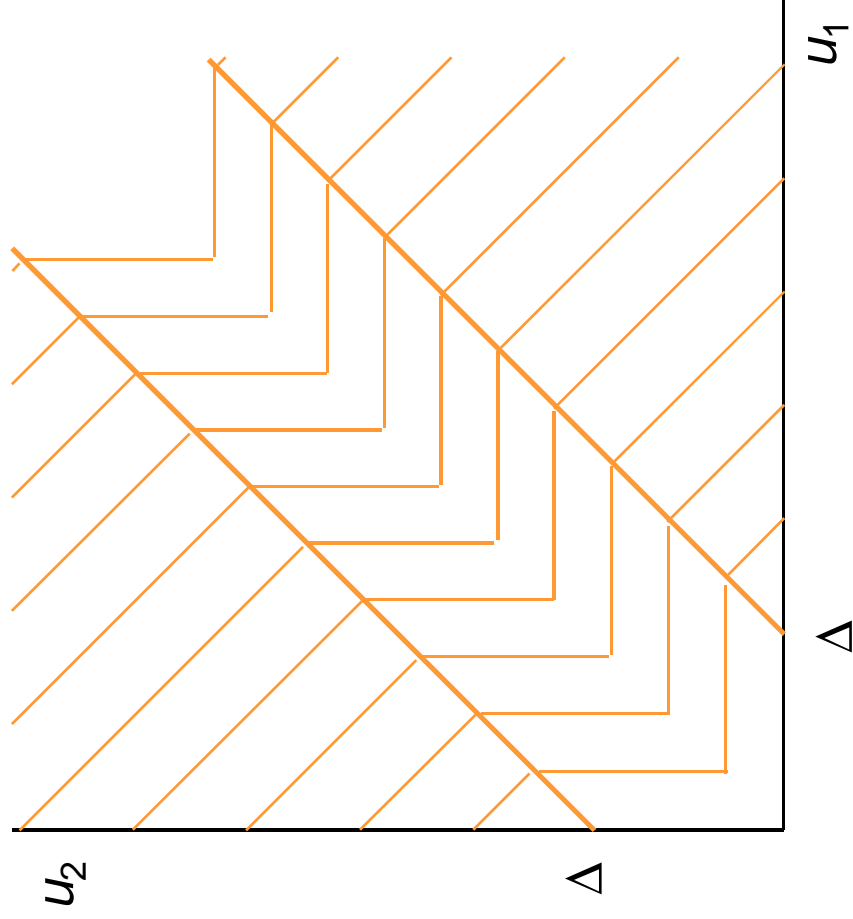


# Combining Equity and Efficiency

- In particular:
  - Switch from **Rawlsian** to **utilitarian** when **inequality** exceeds  $\Delta$ .
  - Build mixed integer programming model.
  - Let  $u_j$  = utility allocated to person  $i$
- For 2 persons:
  - Maximize  $\min_j \{u_1, u_2\}$  (Rawlsian) when  $|u_1 - u_2| \leq \Delta$
  - Maximize  $u_1 + u_2$  (utilitarian) when  $|u_1 - u_2| > \Delta$

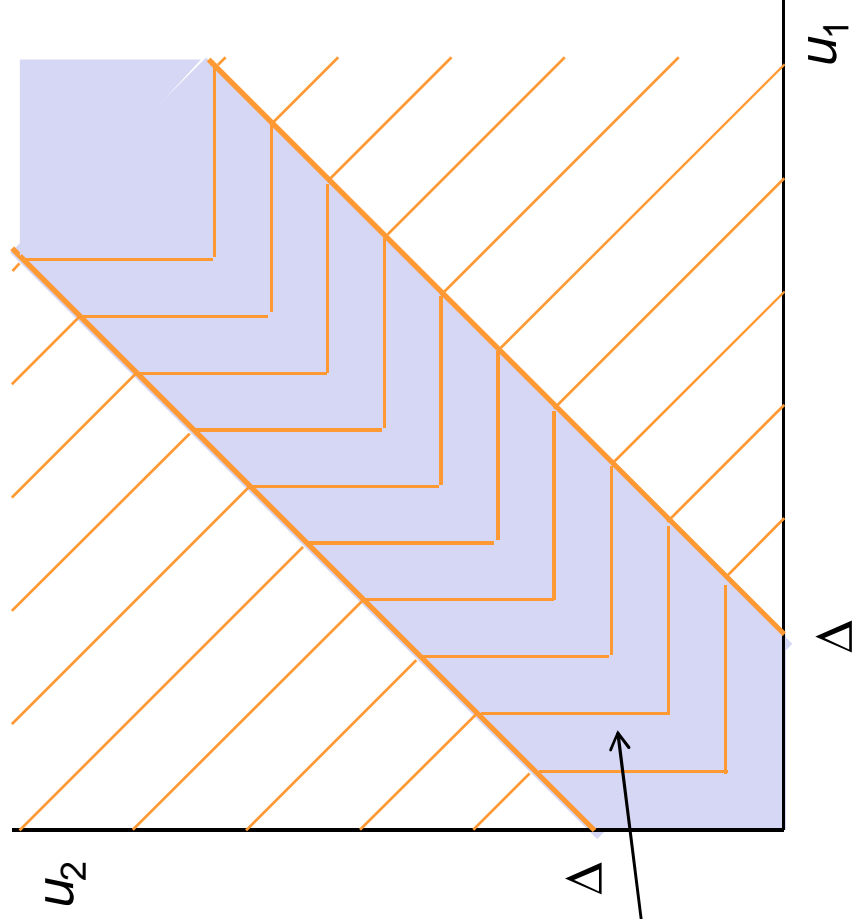
# Two-person Model

Contours of **social welfare function** for 2 persons.



# Two-person Model

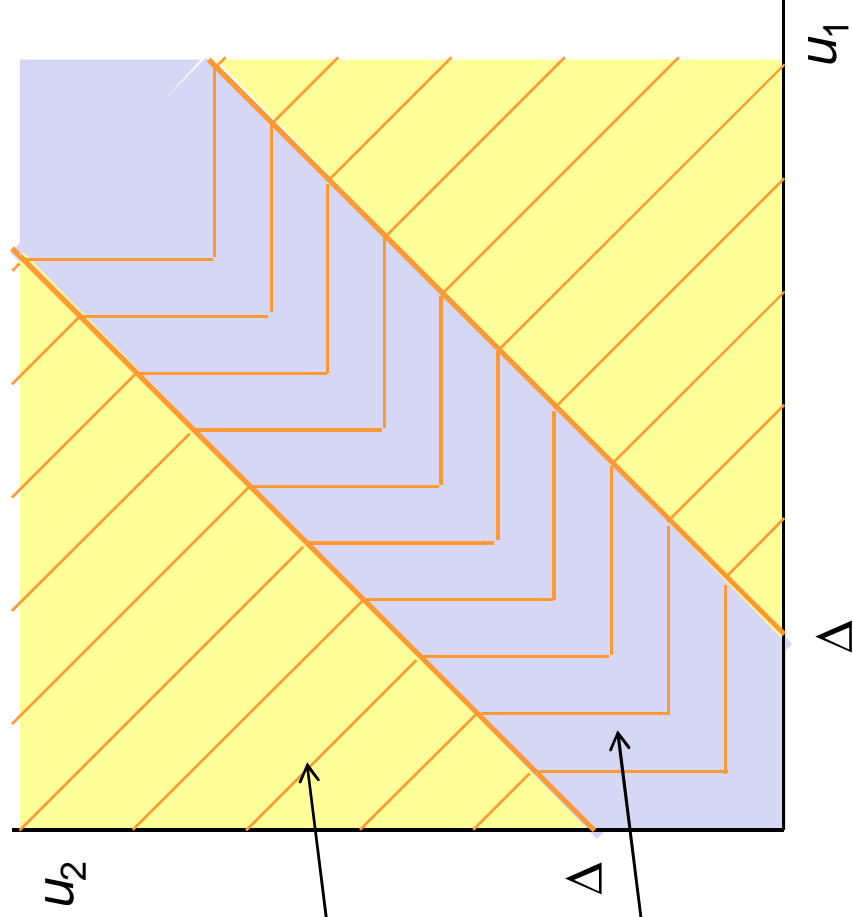
Contours of **social welfare function** for 2 persons.



Rawlsian region  
 $\min\{u_1, u_2\}$

# Two-person Model

Contours of **social welfare function** for 2 persons.



Utilitarian region

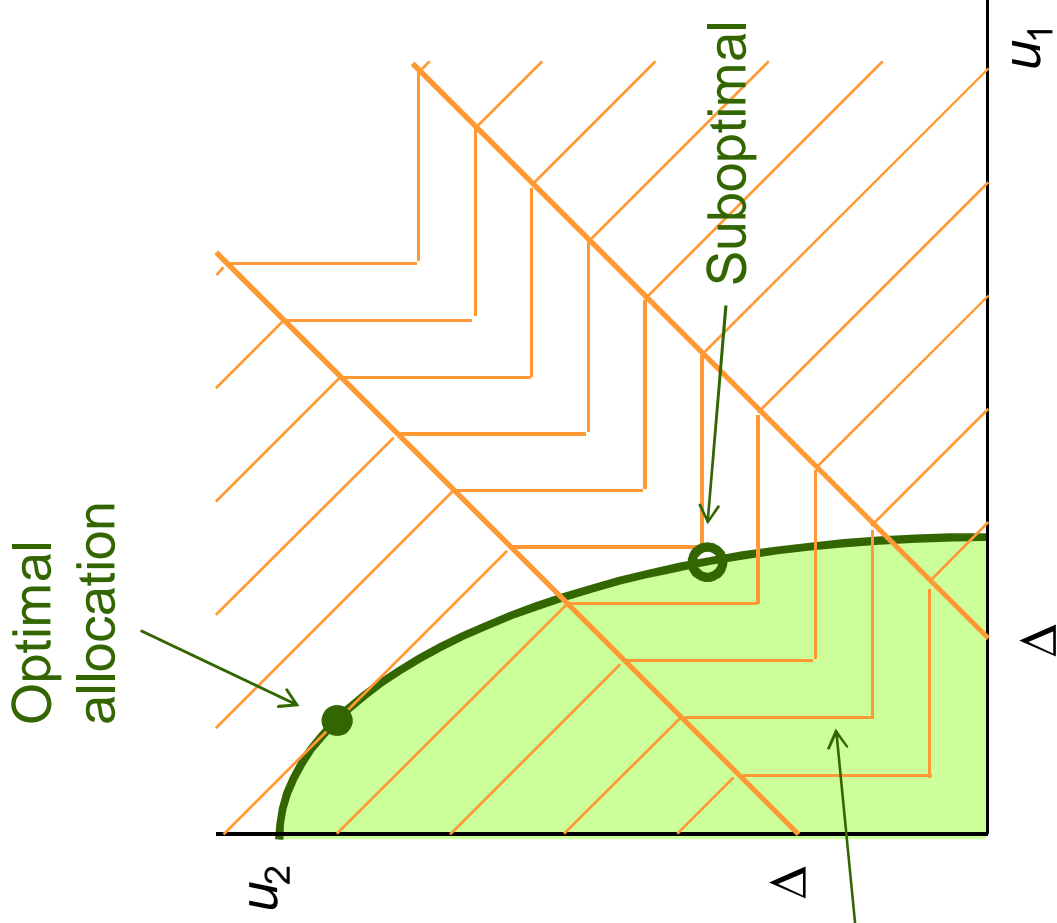
$$u_1 + u_2$$

Rawlsian region

$$\min\{u_1, u_2\}$$

Person 1 is harder to treat.

But maximizing person 1's health requires too much sacrifice from person 2.



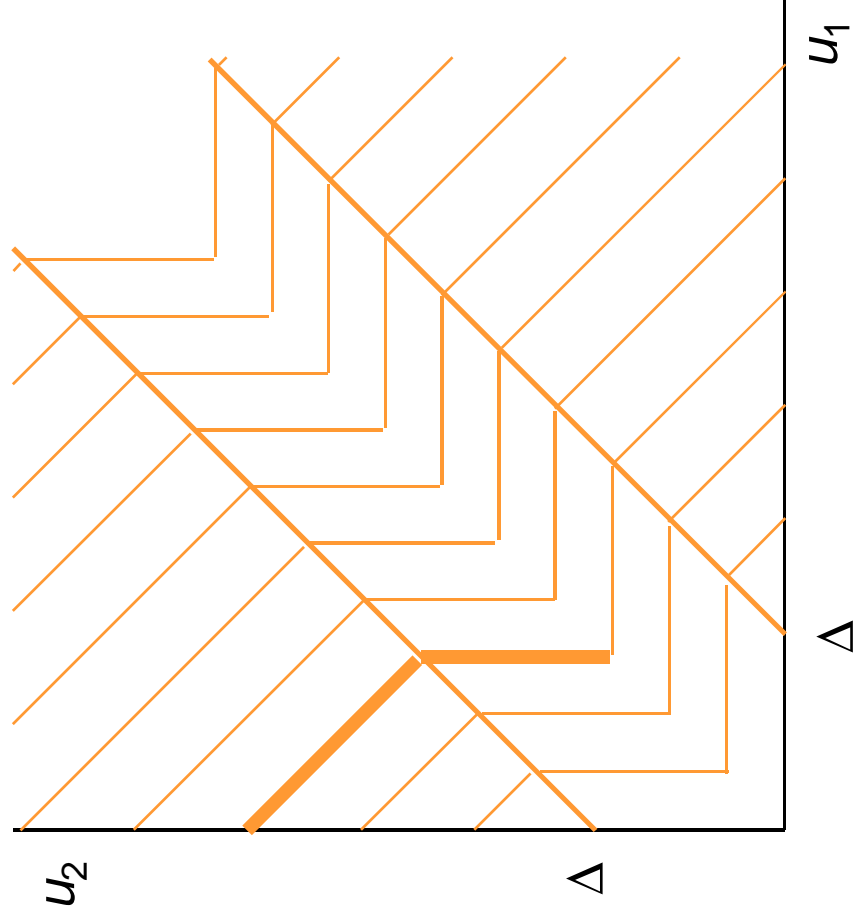
Feasible set

# Advantages

- **Only one parameter  $\Delta$** 
  - **Focus** for debate.
  - $\Delta$  has **intuitive meaning** (unlike weights)
  - Examine **consequences** of different settings for  $\Delta$
  - Find **least objectionable** setting
  - Results in a **consistent** policy

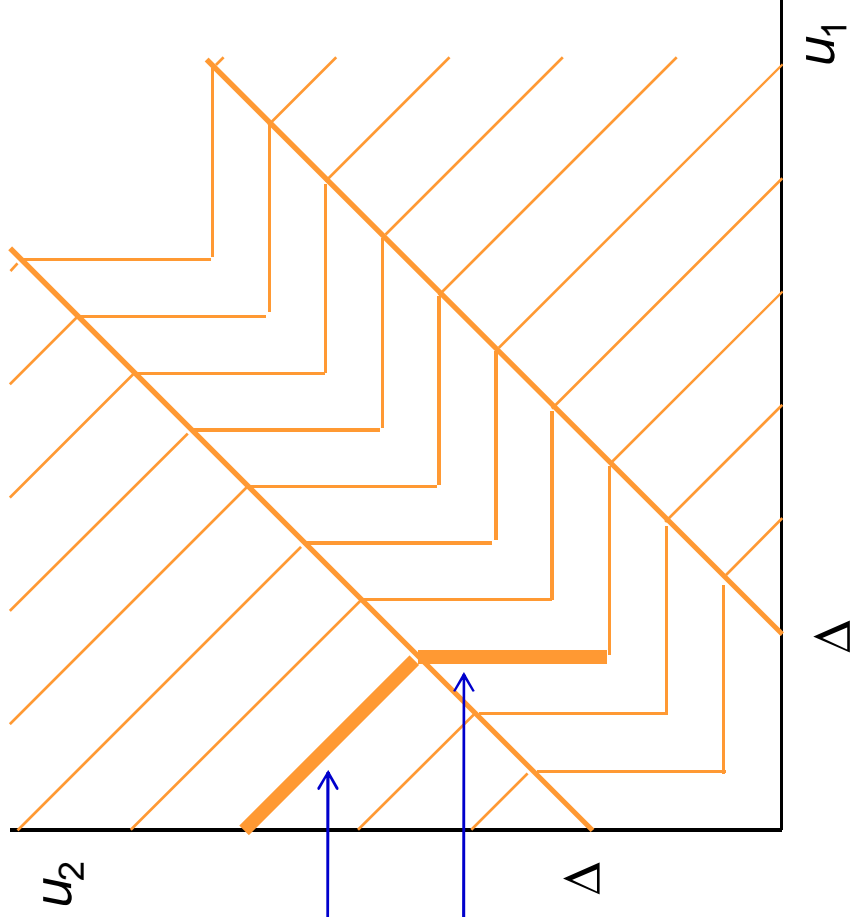
# Social Welfare Function

We want continuous contours...



# Social Welfare Function

We want continuous contours...



$$u_1 + u_2$$

$$2\min\{u_1, u_2\} + \Delta$$

So we use affine transform of Rawlsian criterion



# Social Welfare Function

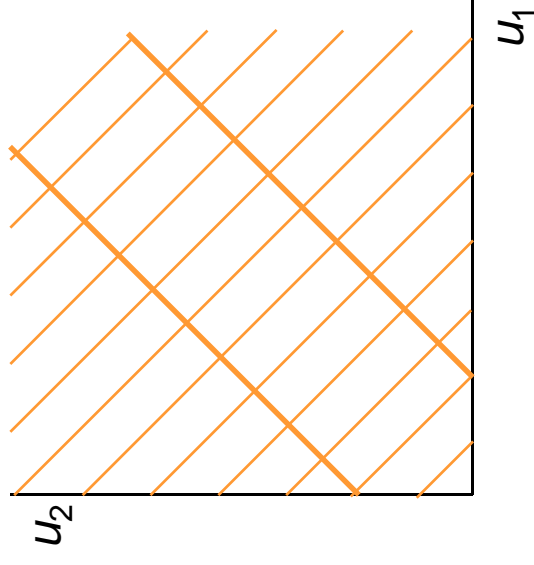
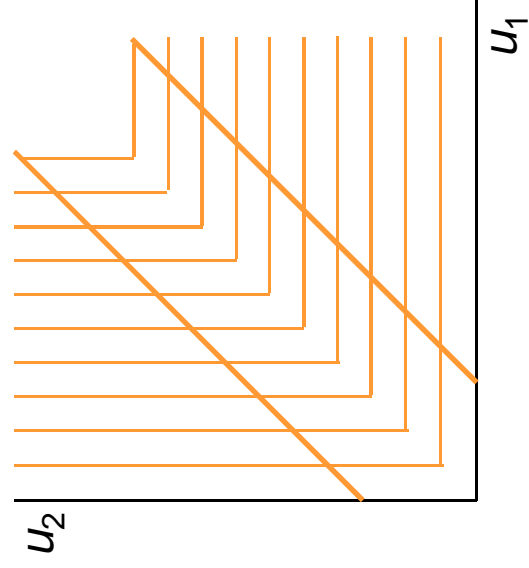
The social welfare problem becomes

$$\max z$$
$$z \leq \begin{cases} 2\min\{u_1, u_2\} + \Delta, & \text{if } |u_1 - u_2| \leq \Delta \\ u_1 + u_2, & \text{otherwise} \end{cases}$$

constraints on feasible set

# MILP Model

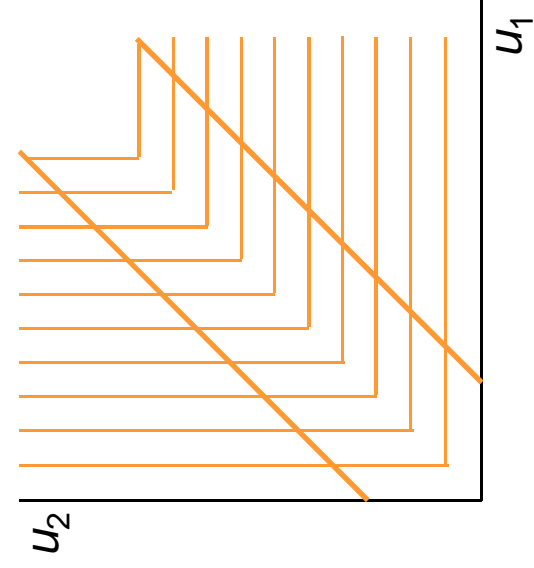
Epigraph is union of 2 polyhedra.



# MILP Model

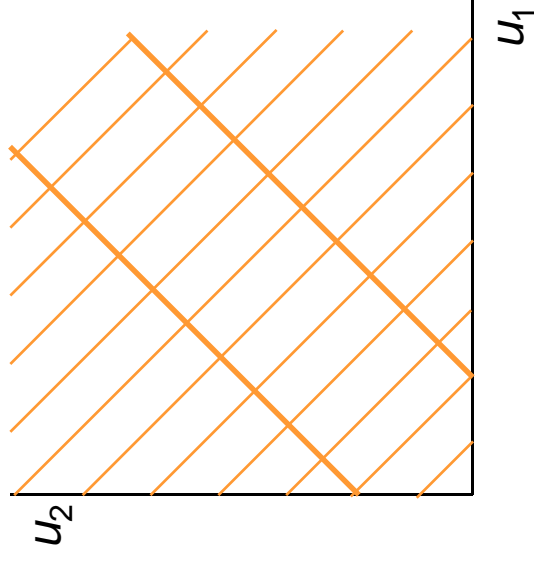
Epigraph is union of 2 polyhedra.

Because they have **different recession cones**, there is no MILP model.



Recession directions  $(u_1, u_2, z)$

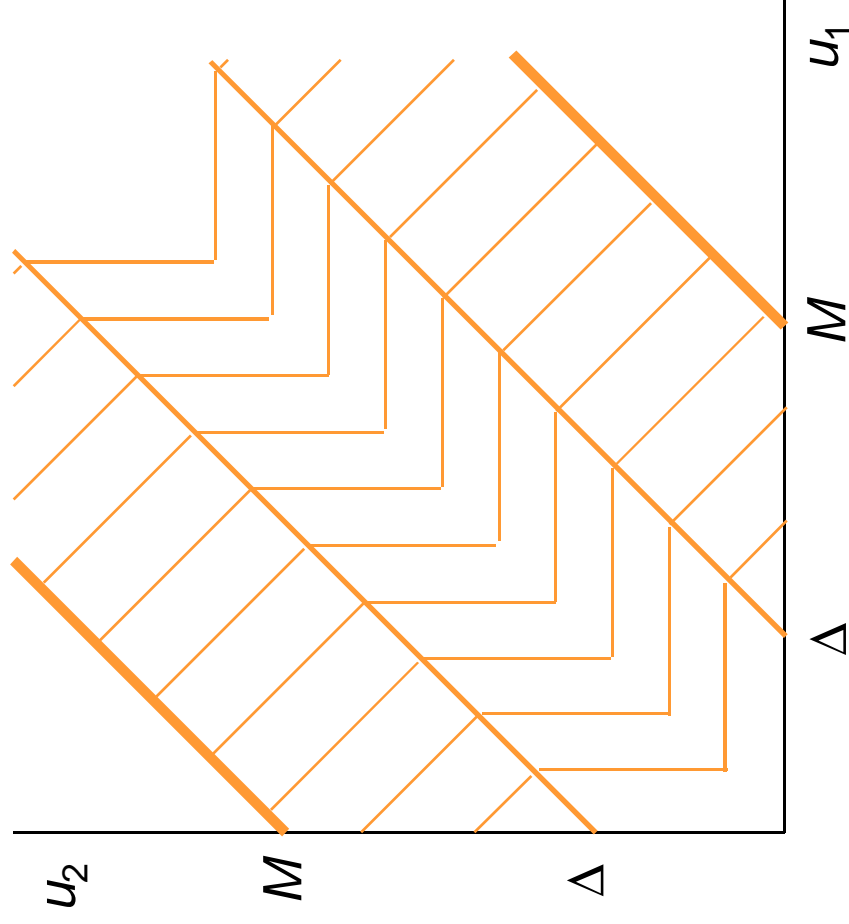
$(0,1,0)$   $(1,1,2)$   $(1,0,0)$



$(0,1,1)$   $(1,0,1)$

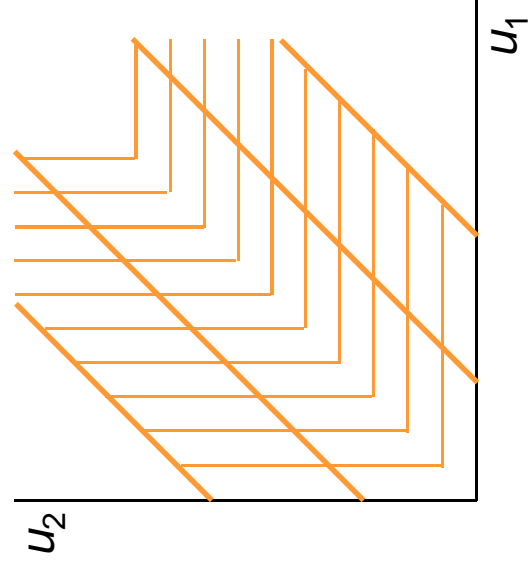
# MILP Model

Impose constraints  $|u_1 - u_2| \leq M$



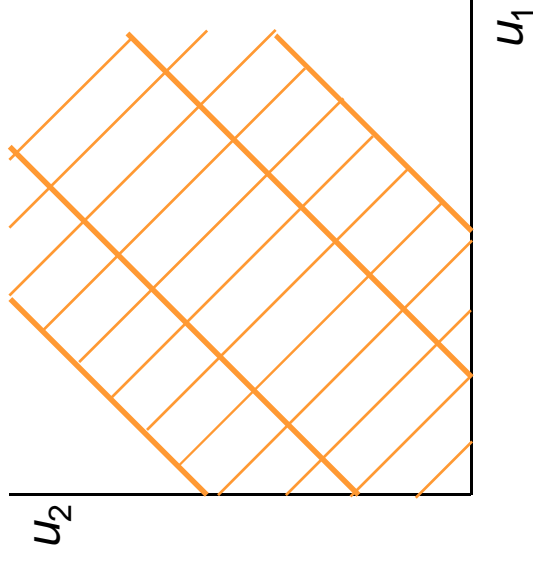
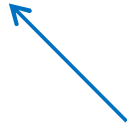
# MILP Model

This equalizes recession cones.



Recession directions  
 $(u_1, u_2, z)$

$(1, 1, 2)$



$(1, 1, 2)$



# MILP Model

We have the model...

max  $z$

$$z \leq 2u_i + \Delta + (M - \Delta)\delta, \quad i=1,2$$

$$z \leq u_1 + u_2 + \Delta(1 - \delta)$$

$$u_1 - u_2 \leq M, \quad u_2 - u_1 \leq M$$

$$u_1, u_2 \geq 0$$

$$\delta \in \{0,1\}$$

$u_1$

constraints on feasible set

# MILP Model

We have the model...

$$\begin{aligned} & \max z \\ & z \leq 2u_i + \Delta + (M - \Delta)\delta, \quad i=1,2 \\ & z \leq u_1 + u_2 + \Delta(1 - \delta) \\ & u_1 - u_2 \leq M, \quad u_2 - u_1 \leq M \\ & u_1, u_2 \geq 0 \\ & \delta \in \{0,1\} \end{aligned}$$

$u_1$

This is a **convex hull** formulation.

# ***n*-person Model**

Rewrite the 2-person social welfare function as...

$$\Delta + 2u_{\min} + (u_1 - u_{\min} - \Delta)^+ + (u_2 - u_{\min} - \Delta)^+ \quad \alpha^+ = \max\{0, \alpha\}$$

$\min\{u_1, u_2\}$

$u_1$



## ***n*-person Model**

Rewrite the 2-person social welfare function as...

$$\Delta + 2u_{\min} + (u_1 - u_{\min} - \Delta)^+ + (u_2 - u_{\min} - \Delta)^+ \quad \alpha^+ = \max\{0, \alpha\}$$

$\min\{u_1, u_2\}$

This can be generalized to  $n$  persons:

$$(n-1)\Delta + nu_{\min} + \sum_{j=1}^n (u_j - u_{\min} - \Delta)^+ \quad u_1$$

## **$n$ -person Model**

Rewrite the 2-person social welfare function as...

$$\min\{u_1, u_2\} \left[ \Delta + 2u_{\min} + (u_1 - u_{\min} - \Delta)^+ + (u_2 - u_{\min} - \Delta)^+ \right] \quad \alpha^+ = \max\{0, \alpha\}$$

This can be generalized to  $n$  persons:

$$(n-1)\Delta + nu_{\min} + \sum_{j=1}^n (u_j - u_{\min} - \Delta)^+ \quad u_1$$

Epigraph is a union of  $n!$  polyhedra with same recession direction  
 $(u, z) = (1, \dots, 1, n)$  if we require  $|u_i - u_j| \leq M$

So there is an MILP model...

## ***n*-person MILP Model**

To avoid  $n!$  0-1 variables, add auxiliary variables  $w_{ij}$

$$\begin{aligned} \max z \\ z &\leq u_i + \sum_{j \neq i} w_{ij}, \text{ all } i \\ w_{ij} &\leq \Delta + u_i + \delta_{ij}(M - \Delta), \text{ all } i, j \text{ with } i \neq j \\ w_{ij} &\leq u_j + (1 - \delta_{ij})\Delta, \text{ all } i, j \text{ with } i \neq j \\ u_i - u_j &\leq M, \text{ all } i, j \\ u_i &\geq 0, \text{ all } i \\ \delta_{ij} &\in \{0, 1\}, \text{ all } i, j \text{ with } i \neq j \end{aligned}$$

## **$n$ -person MILP Model**

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**Theorem.** The model is correct (not easy to prove).

## **$n$ -person MILP Model**

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**Theorem.** The model is correct (not easy to prove).

**Theorem.** This is a convex hull formulation (not easy to prove).

## ***n*-group Model**

In practice, funds may be allocated to groups of different sizes

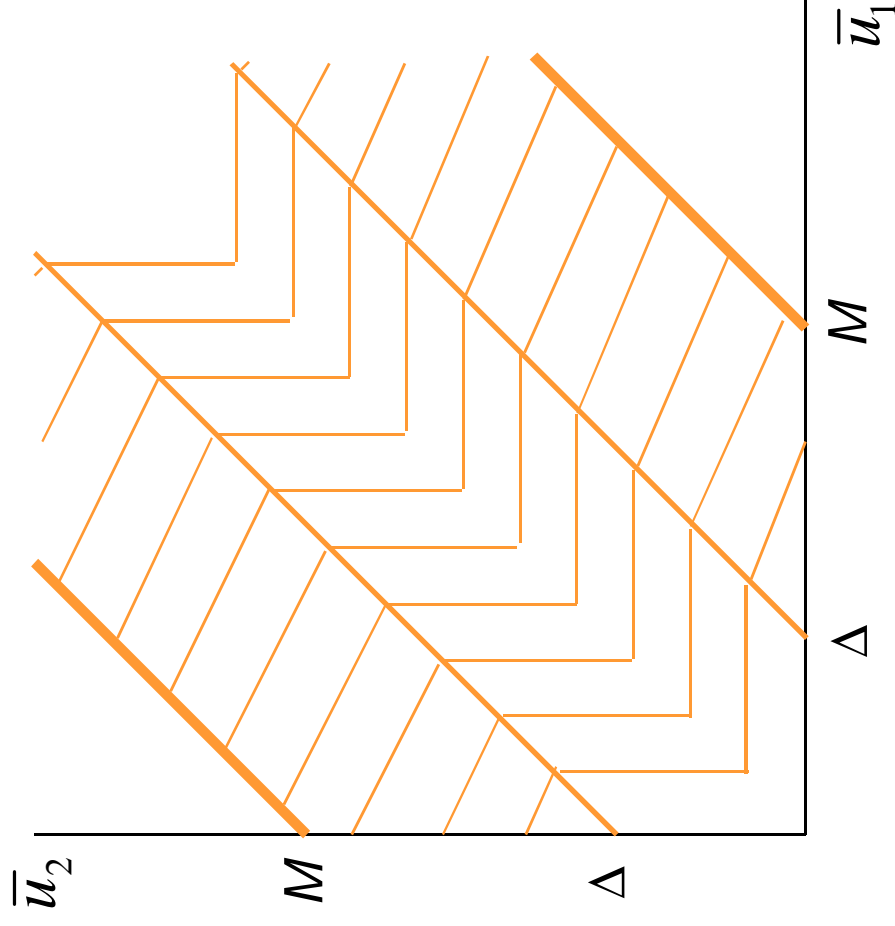
For example, disease/treatment categories.

Let  $\bar{u}_i$  = average utility gained by a person in group  $i$

$n_i$  = size of group  $i$

# ***n*-group Model**

2-person case with  $n_1 < n_2$ . Contours have slope  $-n_1/n_2$



## ***n*-group MILP Model**

Again add auxiliary variables  $w_{ij}$

$$\begin{aligned} \max z \\ z &\leq (n_i - 1)\Delta + n_i \bar{u}_i + \sum_{j \neq i} w_{ij}, \text{ all } i \\ w_{ij} &\leq n_j (\bar{u}_i + \Delta) + \delta_{ij} n_j (M - \Delta), \text{ all } i, j \text{ with } i \neq j \\ w_{ij} &\leq \bar{u}_j + (1 - \delta_{ij}) n_j \Delta, \text{ all } i, j \text{ with } i \neq j \\ \bar{u}_i - \bar{u}_j &\leq M, \text{ all } i, j \\ \bar{u}_i &\geq 0, \text{ all } i \\ \delta_{ij} &\in \{0, 1\}, \text{ all } i, j \text{ with } i \neq j \end{aligned}$$

$u_1$

**Theorem.** The model is correct.

**Theorem.** This is a convex hull formulation.



## Health Example

Measure utility in QALYs (quality-adjusted life years).

QALY and cost data based on Briggs & Gray, (2000) etc.

Each group is a disease/treatment pair.

Treatments are discrete, so group funding is all-or-nothing.

Divide groups into relatively homogeneous subgroups.

$u_1$

# Health Example

Add constraints to define feasible set...

max  $z$

$$z \leq (n_i - 1)\Delta + n_i \bar{u}_i + \sum_{j \neq i} w_{ij}, \text{ all } i$$

$$w_{ij} \leq n_j (\bar{u}_i + \Delta) + \delta_{ij} n_j (M - \Delta), \text{ all } i, j \text{ with } i \neq j$$

$$w_{ij} \leq \bar{u}_j + (1 - \delta_{ij}) n_j \Delta, \text{ all } i, j \text{ with } i \neq j$$

$$\bar{u}_i - \bar{u}_j \leq M, \text{ all } i, j$$

$$\bar{u}_i \geq 0, \text{ all } i$$

$$\delta_{ij} \in \{0, 1\}, \text{ all } i, j \text{ with } i \neq j$$

$u_1$

$y_i$  indicates whether group  $i$  is funded

$$\bar{u}_i = q_i y_i + \alpha_i$$

$$\sum_i n_i c_i y_i \leq \text{budget}$$

$$y_i \in \{0, 1\}, \text{ all } i$$

# QALY & cost data

## Part 1

Intervention	Cost per person $c_i$ (£)	QALYs gained $q_i$	Cost per QALY (£)	QALYs without intervention $\alpha_i$	Subgroup size $n_i$
<i>Pacemaker for atrioventricular heart block</i>					
Subgroup A	3500	3	1167	13	35
Subgroup B	3500	5	700	10	45
Subgroup C	3500	10	350	5	35
<i>Hip replacement</i>					
Subgroup A	3000	2	1500	3	45
Subgroup B	3000	4	750	4	45
Subgroup C	3000	8	375	5	45
<i>Valve replacement for aortic stenosis</i>					
Subgroup A	4500	3	1500	2.5	20
Subgroup B	4500	5	900	3	20
Subgroup C	4500	10	450	3.5	20
<i>CABG<sup>1</sup> for left main disease</i>					
Mild angina	3000	1.25	2400	4.75	50
Moderate angina	3000	2.25	1333	3.75	55
Severe angina	3000	2.75	1091	3.25	60
<i>CABG for triple vessel disease</i>					
Mild angina	3000	0.5	6000	5.5	50
Moderate angina	3000	1.25	2400	4.75	55
Severe angina	3000	2.25	1333	3.75	60
<i>CABG for double vessel disease</i>					
Mild angina	3000	0.25	12,000	5.75	60
Moderate angina	3000	0.75	4000	5.25	65
Severe angina	3000	1.25	2400	4.75	70

# QALY & cost data

## Part 2

Intervention	Cost per person $c_i$ (£)	QALYs gained $q_i$	Cost per QALY (£)	QALYs without intervention $\alpha_i$	Subgroup size $n_i$
<i>Kidney transplant</i>	22,500	4.5	5000	1.1	2
Subgroup A	15,000	4	3750	1	8
Subgroup B	15,000	6	2500	1	8
<i>Kidney dialysis</i>					
<i>Less than 1 year survival</i>					
Subgroup A	5000	0.1	50,000	0.3	8
<i>1-2 years survival</i>					
Subgroup B	12,000	0.4	30,000	0.6	6
<i>2-5 years survival</i>					
Subgroup C	20,000	1.2	16,667	0.5	4
Subgroup D	28,000	1.7	16,471	0.7	4
Subgroup E	36,000	2.3	15,652	0.8	4
<i>5-10 years survival</i>					
Subgroup F	46,000	3.3	13,939	0.6	3
Subgroup G	56,000	3.9	14,359	0.8	2
Subgroup H	66,000	4.7	14,043	0.9	2
Subgroup I	77,000	5.4	14,259	1.1	2
<i>At least 10 years survival</i>					
Subgroup J	88,000	6.5	13,538	0.9	2
Subgroup K	100,000	7.4	13,514	1.0	1
Subgroup L	111,000	8.2	13,537	1.2	1

# Results

Total budget £3 million

Δ range	Pace-maker	Hip repl.	Aortic valve	CABG			Heart trans.	Kidney trans.	Kidney dialysis				
				L	3	2			< 1	1-2	2-5	5-10	> 10
0-3.3	111	111	111	111	111	111	1	11	0	0	000	0000	000
3.4-4.0	111	111	111	111	111	111	0	11	1	0	000	0000	000
4.0-4.4	111	111	111	111	111	111	0	01	1	0	000	0000	001
4.5-5.01	111	011	111	111	111	111	1	01	1	0	000	0000	011
5.02-5.55	111	011	011	111	111	111	0	01	1	0	000	0001	011
5.56-5.58	111	011	011	111	111	011	0	01	1	0	000	0001	111
5.59	111	011	011	110	111	111	0	01	1	0	000	0001	111
5.60-13.1	111	111	111	101	000	000	1	11	1	0	111	1111	111
13.2-14.2	111	011	111	011	000	000	1	11	1	1	111	1111	111
14.3-15.4	111	111	111	011	000	000	1	11	1	1	101	1111	111
15.5-up	111	011	111	011	001	000	1	11	1	0	011	1111	111

# Results

Utilitarian solution

$\Delta$ range	Pace-maker	Hip repl.	Aortic valve	L	CABG	Heart trans.	Kidney trans.	Kidney dialysis
				3	2	< 1	1-2	5-10
							2-5	> 10
0-3.3	111	111	111	111	111	1	11	0 0 000 0000 000
3.4-4.0	111	111	111	111	111	0	11	1 0 000 0000 000
4.0-4.4	111	111	111	111	111	0	01	1 0 000 0000 001
4.5-5.01	111	011	111	111	111	1	01	1 0 000 0000 011
5.02-5.55	111	011	011	111	111	0	01	1 0 000 0001 011
5.56-5.58	111	011	011	111	111	0	01	1 0 000 0001 111
5.59	111	011	011	110	111	0	01	1 0 000 0001 111
5.60-13.1	111	111	111	101	000 000	1	11	1 0 111 1111 111
13.2-14.2	111	011	111	011	000 000	1	11	1 1 111 1111 111
14.3-15.4	111	111	111	011	000 000	1	11	1 1 101 1111 111
15.5-up	111	011	111	011	001 000	1	11	1 0 011 1111 111

# Results

Rawlsian solution

$\Delta$ range	Pace- maker	Hip repl.	Aortic valve	L	CABG	Heart trans.	Kidney trans.	Kidney dialysis
				3	2	< 1	1-2	5-10
							2-5	> 10
0-3.3	111	111	111	111	111	1	11	0 0 000 0000 000
3.4-4.0	111	111	111	111	111	0	11	1 0 000 0000 000
4.0-4.4	111	111	111	111	111	0	01	1 0 000 0000 001
4.5-5.01	111	011	111	111	111	1	01	1 0 000 0000 011
5.02-5.55	111	011	011	111	111	0	01	1 0 000 0001 011
5.56-5.58	111	011	011	111	111	0	01	1 0 000 0001 111
5.59	111	011	011	110	111	0	01	1 0 000 0001 111
5.60-13.1	111	111	111	101	000	000	11	1 0 111 1111 111
13.2-14.2	111	011	111	011	000	000	11	1 1 111 1111 111
14.3-15.4	111	111	111	011	000	000	11	1 1 101 1111 111
15.5-up	111	011	111	011	001	000	11	1 0 011 1111 111



# Results

Fund for all  $\Delta$



$\Delta$ range	Pace- maker repl.	Hip repl.	Aortic valve	CABG			Heart trans.			Kidney trans.			Kidney dialysis		
				L	3	2	1	2	1	1-2	2-5	5-10	> 10		
0-3.3	111	111	111	111	111	111	1	11	11	0	0	0	000	0000	000
3.4-4.0	111	111	111	111	111	111	0	11	11	1	0	0	000	0000	000
4.0-4.4	111	111	111	111	111	111	0	01	01	1	0	0	000	0000	001
4.5-5.01	111	011	111	111	111	111	1	01	01	1	0	0	000	0000	011
5.02-5.55	111	011	011	111	111	111	0	01	01	1	0	0	000	0001	011
5.56-5.58	111	011	011	111	111	011	0	01	01	1	0	0	000	0001	111
5.59	111	011	011	110	111	111	0	01	01	1	0	0	000	0001	111
5.60-13.1	111	111	111	101	000	000	1	11	11	1	0	0	111	1111	111
13.2-14.2	111	011	111	011	000	000	1	11	11	1	1	1	111	1111	111
14.3-15.4	111	111	111	011	000	000	1	11	11	1	1	1	101	1111	111
15.5-up	111	011	111	011	001	000	1	11	11	1	0	0	011	1111	111



# Results

More dialysis with larger  $\Delta$ , beginning with longer life span

$\Delta$ range	Pace-maker	Hip repl.	Aortic valve	CABG			Heart trans.	Kidney trans.	Kidney dialysis				
				L	3	2			< 1	1-2	2-5	5-10	> 10
0-3.3	111	111	111	111	111	111	1	11	0	0	000	0000	000
3.4-4.0	111	111	111	111	111	111	0	11	1	0	000	0000	000
4.0-4.4	111	111	111	111	111	111	0	01	1	0	000	0000	001
4.5-5.01	111	011	111	111	111	111	1	01	1	0	000	0000	011
5.02-5.55	111	011	011	111	111	111	0	01	1	0	000	0001	011
5.56-5.58	111	011	011	111	111	011	0	01	1	0	000	0001	111
5.59	111	011	011	110	111	111	0	01	1	0	000	0001	111
5.60-13.1	111	111	111	101	000	000	1	11	1	0	111	1111	111
13.2-14.2	111	011	111	011	000	000	1	11	1	1	111	1111	111
14.3-15.4	111	111	111	011	000	000	1	11	1	1	101	1111	111
15.5-up	111	011	111	011	001	000	1	11	1	0	011	1111	111

# Results

Abrupt change at  $\Delta = 5.60$

$\Delta$ range	Pace- maker	Hip repl.	Aortic valve	L	CABG			Heart trans.	Kidney			Kidney dialysis	
					3	2	trans.		< 1	1-2	2-5		5-10
0-3.3	111	111	111	111	111	111	1	11	0	0	000	0000	000
3.4-4.0	111	111	111	111	111	111	0	11	1	0	000	0000	000
4.0-4.4	111	111	111	111	111	111	0	01	1	0	000	0000	001
4.5-5.01	111	011	111	111	111	111	1	01	1	0	000	0000	011
5.02-5.55	111	011	011	111	111	111	0	01	1	0	000	0001	011
5.56-5.58	111	011	011	111	111	011	0	01	1	0	000	0001	111
5.59	111	011	011	110	111	111	0	01	1	0	000	0001	111
5.60-13.1	111	111	111	101	000	000	1	11	1	0	111	1111	111
13.2-14.2	111	011	111	011	000	000	1	11	1	1	111	1111	111
14.3-15.4	111	111	111	011	000	000	1	11	1	1	101	1111	111
15.5-up	111	011	111	011	001	000	1	11	1	0	011	1111	111

# Results

Come and go together

$\Delta$ range	Pace- maker	Hip repl.	Aortic valve	L	CABG		Heart trans.	Kidney		Kidney dialysis			
					3	2		< 1	1-2	2-5	5-10	> 10	
0-3.3	111	111	111	111	111	111	1	11	0	0	000	0000	000
3.4-4.0	111	111	111	111	111	111	0	11	1	0	000	0000	000
4.0-4.4	111	111	111	111	111	111	0	01	1	0	000	0000	001
4.5-5.01	111	011	111	111	111	111	1	01	1	0	000	0000	011
5.02-5.55	111	011	011	111	111	111	0	01	1	0	000	0001	011
5.56-5.58	111	011	011	111	111	111	0	01	1	0	000	0001	111
5.59	111	011	011	110	111	111	0	01	1	0	000	0001	111
5.60-13.1	111	111	111	101	000	000	1	11	1	0	111	1111	111
13.2-14.2	111	011	111	011	000	000	1	11	1	1	111	1111	111
14.3-15.4	111	111	111	011	000	000	1	11	1	1	101	1111	111
15.5-up	111	011	111	011	001	000	1	11	1	0	011	1111	111

# Results

In-out-in

$\Delta$ range	Pace- maker	Hip repl.	Aortic valve	CABG			Heart trans.	Kidney trans.	Kidney dialysis				
				L	3	2			< 1	1-2	2-5	5-10	> 10
0-3.3	111	111	111	111	111	111	1	11	0	0	000	0000	000
3.4-4.0	111	111	111	111	111	111	0	11	1	0	000	0000	000
4.0-4.4	111	111	111	111	111	111	0	01	1	0	000	0000	001
4.5-5.01	111	011	111	111	111	111	1	01	1	0	000	0000	011
5.02-5.55	111	011	011	111	111	111	0	01	1	0	000	0001	011
5.56-5.58	111	011	011	111	111	011	0	01	1	0	000	0001	111
5.59	111	011	011	110	111	111	0	01	1	0	000	0001	111
5.60-13.1	111	111	111	101	000	000	1	11	1	0	111	1111	111
13.2-14.2	111	011	111	011	000	000	1	11	1	1	111	1111	111
14.3-15.4	111	111	111	011	000	000	1	11	1	1	101	1111	111
15.5-up	111	011	111	011	001	000	1	11	1	0	011	1111	111

# Results

Most rapid change. Possible range for politically acceptable compromise

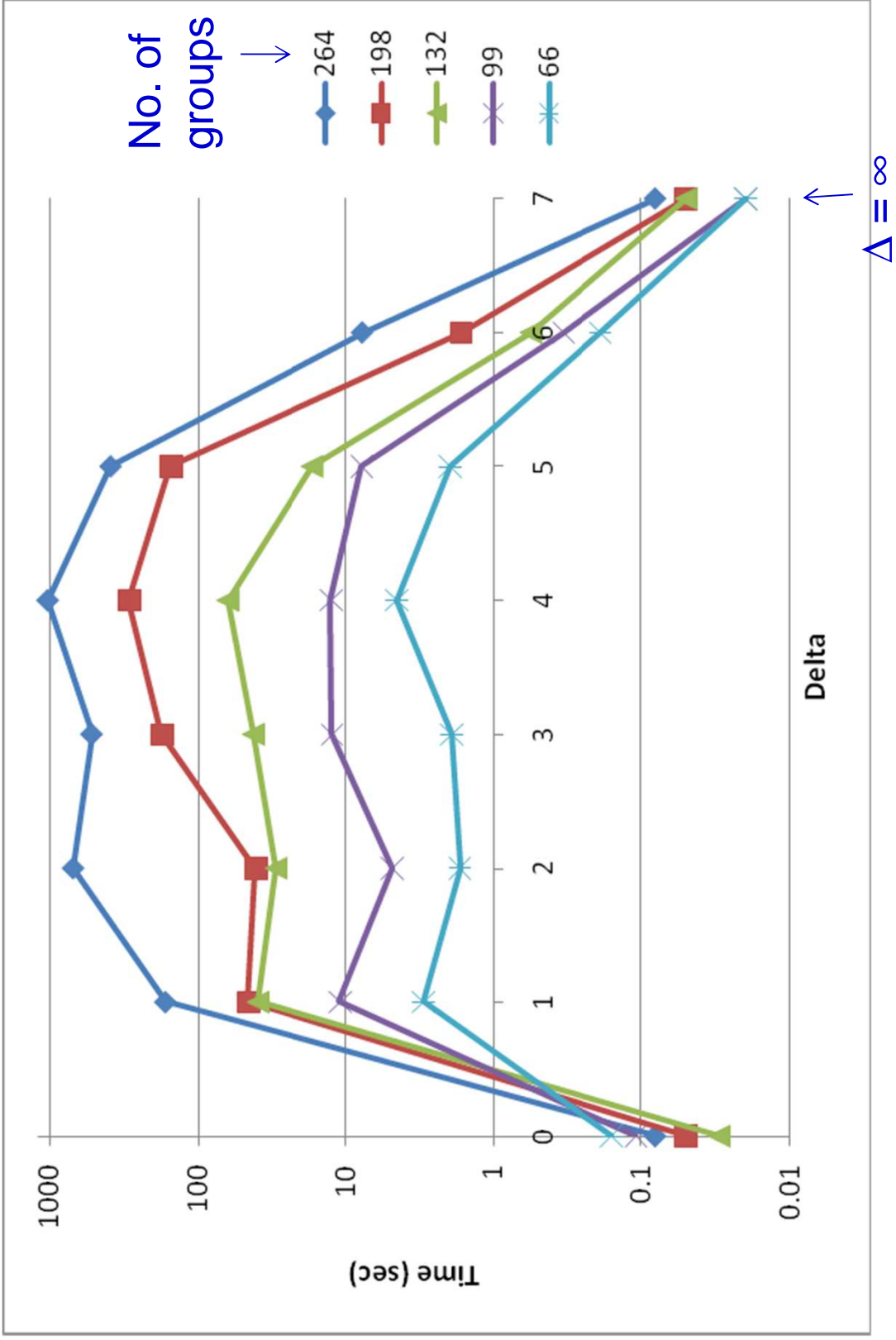
$\Delta$ range	Pace- maker	Hip repl.	Aortic valve	CABG			Heart trans.	Kidney trans.	Kidney dialysis				
				L	3	2			< 1	1-2	2-5	5-10	> 10
0-3.3	111	111	111	111	111	111	1	11	0	0	000	0000	000
3.4-4.0	111	111	111	111	111	111	0	11	1	0	000	0000	000
4.0-4.4	111	111	111	111	111	111	0	01	1	0	000	0000	001
4.5-5.01	111	011	111	111	111	111	1	01	1	0	000	0000	011
5.02-5.55	111	011	011	111	111	111	0	01	1	0	000	0001	011
5.56-5.58	111	011	011	111	111	011	0	01	1	0	000	0001	111
5.59	111	011	011	110	111	111	0	01	1	0	000	0001	111
5.60-13.1	111	111	111	101	000	000	1	11	1	0	111	1111	111
13.2-14.2	111	011	111	011	000	000	1	11	1	1	111	1111	111
14.3-15.4	111	111	111	011	000	000	1	11	1	1	101	1111	111
15.5-up	111	011	111	011	001	000	1	11	1	0	011	1111	111

# Results

32 groups, 1089 integer variables  
 Solution time (CPLEX 12.2) is < 0.5 sec for each  $\Delta$

$\Delta$ range	Pace- maker	Hip repl.	Aortic valve	CABG			Heart trans.	Kidney trans.	Kidney dialysis				
				L	3	2			< 1	1-2	2-5	5-10	> 10
0-3.3	111	111	111	111	111	111	1	11	0	0	000	0000	000
3.4-4.0	111	111	111	111	111	111	0	11	1	0	000	0000	000
4.0-4.4	111	111	111	111	111	111	0	01	1	0	000	0000	001
4.5-5.01	111	011	111	111	111	111	1	01	1	0	000	0000	011
5.02-5.55	111	011	011	111	111	111	0	01	1	0	000	0001	011
5.56-5.58	111	011	011	111	111	011	0	01	1	0	000	0001	111
5.59	111	011	011	110	111	111	0	01	1	0	000	0001	111
5.60-13.1	111	111	111	101	000	000	1	11	1	0	111	1111	111
13.2-14.2	111	011	111	011	000	000	1	11	1	1	111	1111	111
14.3-15.4	111	111	111	011	000	000	1	11	1	1	101	1111	111
15.5-up	111	011	111	011	001	000	1	11	1	0	011	1111	111

# Solution time vs. $\Delta$



# Future Work

- Generalize Rawlsian criterion to  $\text{lexmax}$ .
- Find principled justification for choice of  $\Delta$ .