A Guide to Formulating Equity and Fairness in Optimization Models

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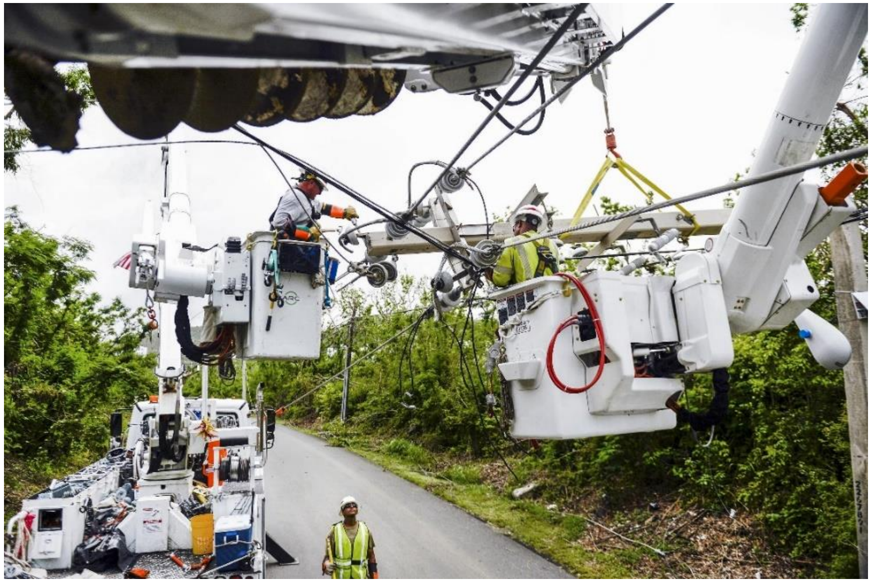
Modeling Fairness

- A growing interest in incorporating **fairness** models, particularly in **public sector** & **humanitarian** settings.
  - Health care resources.
  - Facility location (e.g., emergency services, infrastructure).
  - Disaster recovery (e.g., power restoration)…

Modeling Fairness

- Example: disaster relief
  - Power restoration can focus on **urban** areas first (**efficiency**).
  - This can leave rural areas without power for weeks/months.
  - This happened in Puerto Rico after Hurricane Maria (2017).
- A more **equitable** solution
  - …would give some priority to rural areas without overly sacrificing efficiency.



Modeling Fairness

- It is far from obvious how to formulate equity concerns **mathematically**.
    - Less straightforward than maximizing total benefit or minimizing total cost.
    - Still less obvious how to **combine** equity with total benefit.

Modeling Fairness

- There is **no one** concept of equity or fairness.
    - The appropriate concept **depends on the application**.

Modeling Fairness

- There is **no one** concept of equity or fairness.
  - The appropriate concept **depends on the application**.

- Survey of fairness models, with references:

  V. Chen & J. N. Hooker, A guide to formulating fairness in an optimization model, submitted, 2022.

- Tutorial videos:

  https://cp2021.lirmm.fr/submissions/2001
  http://public.tepper.cmu.edu/jnh/equityINFORMSpgh.pdf

Inequality measures

Criterion	*Linear?*	*Contin?*
Relative range	yes	yes
Relative mean deviation	yes	yes
Coefficient of variation	no	yes
Gini coefficient	yes	yes
Hoover index	yes	yes

Group parity metrics from AI

Criterion	*Linear?*	*Contin?*
Demographic parity	yes	yes
Equalized odds	yes	yes
Predictive rate parity	no	yes

Linear = fairness model introduces only **linear** expressions
Contin. = fairness model introduces only **continuous** variables

Fairness for the disadvantaged

Criterion	*Linear?*	*Contin?*
Maximin (Rawlsian)	yes	yes
Leximax (lexicographic)	yes	yes
McLoone index	yes	no

Combining efficiency & fairness
Convex combinations

Criterion	*Linear?*	*Contin?*
Utility + Gini coefficient	no	yes
Utility * Gini coefficient	yes	yes
Utility + maximin	yes	yes

Linear = fairness model introduces only **linear** expressions
Contin. = fairness model introduces only **continuous** variables

Combining efficiency & fairness
Classical methods

Criterion	*Linear?*	*Contin?*
Alpha fairness	yes	yes
Proportional fairness (Nash bargaining)	yes	yes
Kalai-Smorodinsky bargaining	no	yes

Combining efficiency & fairness
Threshold methods

Criterion	*Linear?*	*Contin?*
Utility + maximin – Utility threshold	yes	no
Utility + maximin – Equity threshold	yes	yes
Utility + leximax – Predefined priorities	yes	no
Utility + leximax – No predefined priorities	yes	no

Linear = fairness model introduces only **linear** expressions
Contin. = fairness model introduces only **continuous** variables

Generic Model

- We formulate each fairness criterion as a **social welfare function (SWF)**.

Individual utilities

$$W(\boldsymbol{u}) = W(u_1, \ldots, u_n)$$

  - Measures desirability of the **magnitude and distribution of utilities** across individuals.
  - **Utility** can be wealth, health, negative cost, etc.
  - **Welfare maximizing**: the SWF becomes the **objective function** of the optimization model.
  - **Welfare constraining**: the SWF imposes a **lower bound** on social welfare (original objective function retained)

Generic Model

Welfare maximizing model

Vector of utilities enjoyed by individuals (or groups)

Vector of resource allocations to individuals (or groups)

$$\max_{\boldsymbol{u},\boldsymbol{x}} \left\{ W(\boldsymbol{u}) \;\middle|\; \boldsymbol{u} = U(\boldsymbol{x}),\; \boldsymbol{x} \in S_{\boldsymbol{x}} \right\}$$

Social welfare function

Vector of utility functions $U_1, \ldots, U_n$

Problem constraints

Generic Model

Welfare constraining model

Vector of utilities enjoyed by individuals (or groups)

Vector of resource allocations to individuals (or groups)

$$\max_{\boldsymbol{u},\boldsymbol{x}} \left\{ f(\boldsymbol{x}) \;\middle|\; \boldsymbol{u} = U(\boldsymbol{x}),\; W(\boldsymbol{u}) \geq \mathrm{LB},\; \boldsymbol{x} \in S_{\boldsymbol{x}} \right\}$$

Original objective function

Vector of utility functions $U_1, \ldots, U_n$

Bound on social welfare

Problem constraints

Inequality Measures

Criterion	*Linear?*	*Contin?*
Relative range	yes	yes
Relative mean deviation	yes	yes
Coefficient of variation	no	yes
Gini coefficient	yes	yes
Hoover index	yes	yes

Inequality Measures

Normally used in welfare constraining models

- To maximize overall efficiency while limiting inequality.

Two views on ethical importance of equality:

Parfit 1997

- **Irreducible:** Inequality is inherently unfair.

Scanlon 2003

- **Reducible:** Inequality is unfair only insofar as it reduces utility.

Frankfurt 2015

All SWFs but one have linear formulations

- Using linear fractional programming.

Inequality Measures

Relative range

$$W(\boldsymbol{u}) = -\frac{u_{\text{max}} - u_{\text{min}}}{\bar{u}}$$

Rationale:

- Perceived inequality is relative to the best off.
- So, move everyone closer to the best off.

Problem:

- Ignores distribution **between** extremes.

Inequality Measures

Relative mean deviation

$$W(\boldsymbol{u}) = -\frac{1}{\bar{u}} \sum_i |u_i - \bar{u}|$$

Rationale:

- Considers all utilities.

Inequality Measures

Coefficient of variation

$$W(\boldsymbol{u}) = -\frac{1}{\bar{u}}\left[\frac{1}{n}\sum_i (u_i - \bar{u})^2\right]^{\frac{1}{2}}$$

Rationale:

- Familiar. Outliers receive extra weight.

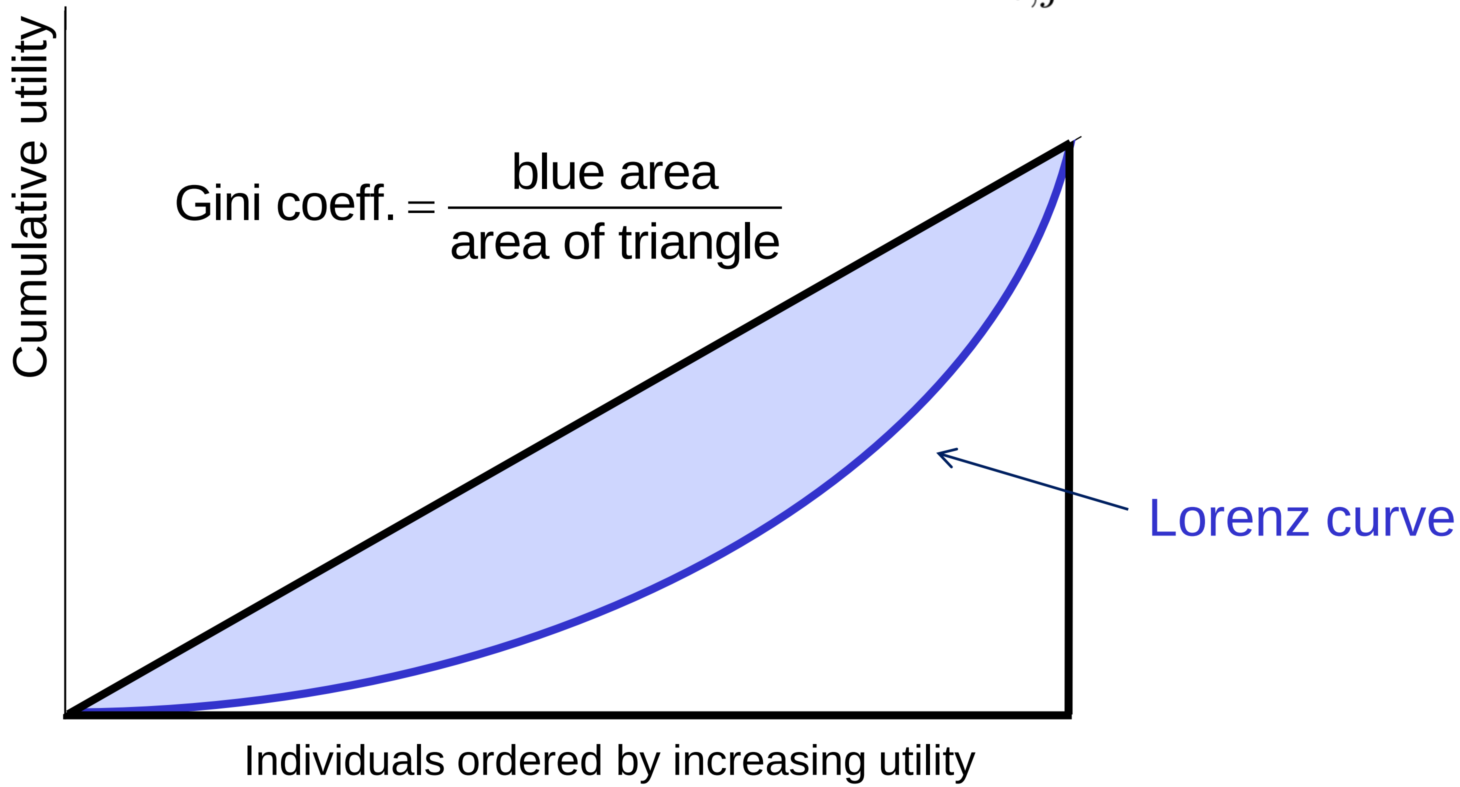
Problem:

- Nonlinear (but convex)

Inequality Measures

Gini coefficient

$$W(\boldsymbol{u}) = -G(\boldsymbol{u}), \quad \text{where } G(\boldsymbol{u}) = \frac{1}{2\bar{u}n^2} \sum_{i,j} |u_i - u_j|$$



Inequality Measures

Gini coefficient

$$W(\boldsymbol{u}) = -G(\boldsymbol{u}), \quad \text{where } G(\boldsymbol{u}) = \frac{1}{2\bar{u}n^2} \sum_{i,j} |u_i - u_j|$$

Rationale:

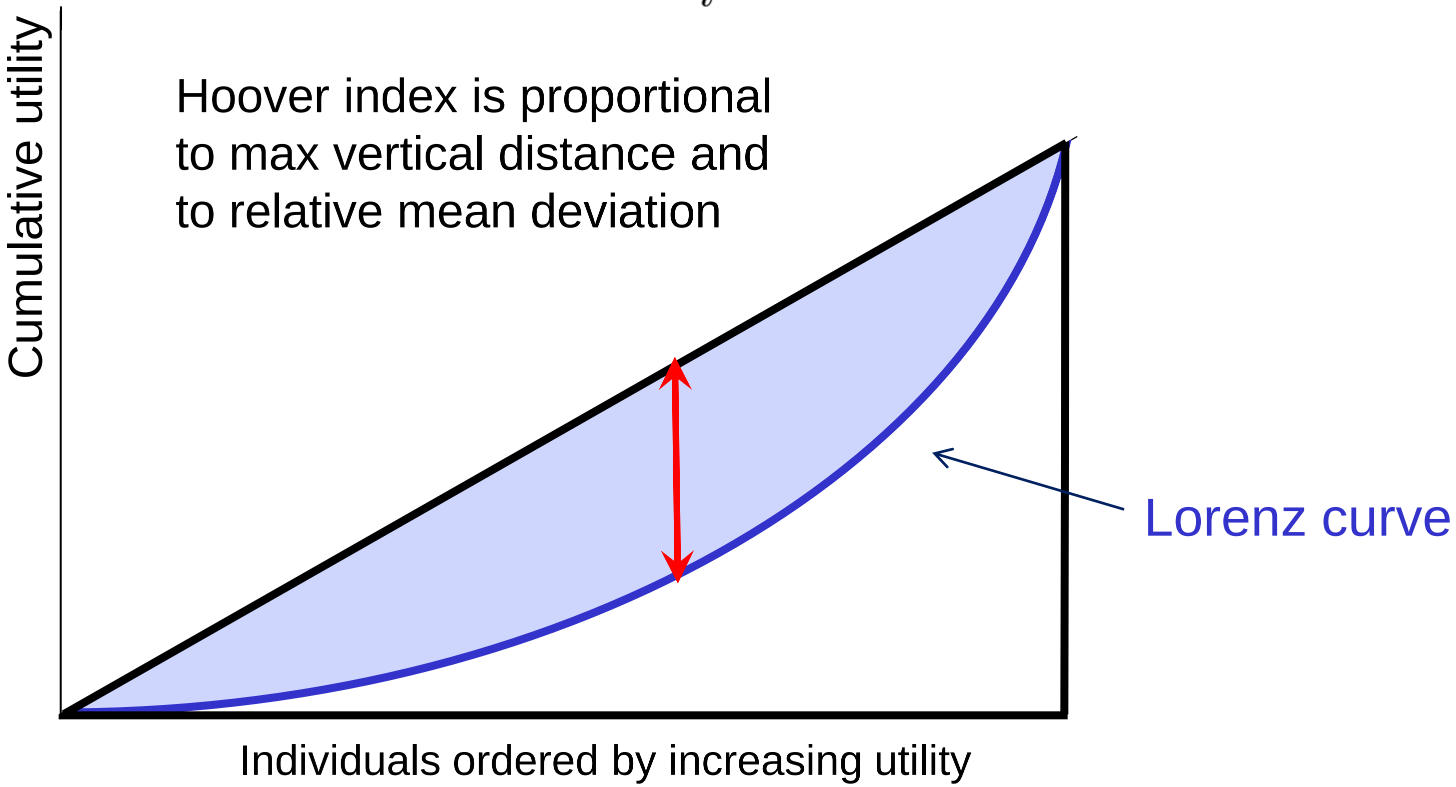
- Relationship to Lorenz curve.
- Widely used.

Inequality Measures

Hoover index

$$W(\boldsymbol{u}) = -\frac{1}{2n\bar{u}} \sum_i |u_i - \bar{u}|$$

Hoover 1936



Inequality Measures

Hoover index

$$W(\boldsymbol{u}) = -\frac{1}{2n\bar{u}} \sum_i |u_i - \bar{u}|$$

Rationale:

- Hoover index is fraction of total utility that would have to be redistributed to achieve perfect equality.
- Same model as relative mean deviation.

Statistical Fairness Metrics

Criterion	*Linear?*	*Contin?*
Demographic parity	yes	yes
Equalized odds	yes	yes
Predictive rate parity	no	yes

Group Parity Metrics

- Widely discussed in **AI**.
- Intended to measure bias against a subgroup
- For example: mortgage loans, job interviews, parole.
- Utility vector $\boldsymbol{u}$ is now vector $\boldsymbol{\delta}$ of yes-no decisions.
- Used in welfare constraining models.

Rationale

- Resonates with popular conceptions of fairness.
- Bias may incur legal problems.

Statistical Fairness Metrics

Notation

- TP = number of true positives (correct yes’s)
- FP = number of false positives (incorrect yes’s).
- TN = number of true negatives (correct no’s).
- FN = number of false negatives (incorrect no’s).

Statistical Fairness Metrics

Demographic parity

- Compare $\dfrac{\mathrm{TP} + \mathrm{FP}}{\mathrm{TP} + \mathrm{TN} + \mathrm{FP} + \mathrm{FN}}$ across 2 groups

$$W(\boldsymbol{\delta}) = 1 - |B(\boldsymbol{\delta})|, \text{ where } B(\boldsymbol{\delta}) = \frac{1}{|N|}\sum_{i \in N} \delta_i - \frac{1}{|N'|}\sum_{i \in N'} \delta_i$$

Majority group

Minority group

Dwork et al. 2012

Rationale

- Equality of outcomes.

Possible problem

- Can discriminate against a minority group that is more qualified than majority group.

Statistical Fairness Metrics

Equalized odds

Equality of opportunity

- Compare $\frac{\text{TP}}{\text{TP} + \text{FN}}$ and $\frac{\text{FP}}{\text{FP} + \text{TN}}$ across 2 groups

$$B(\boldsymbol{\delta}) = \frac{\sum_{i \in N} a_i \delta_i}{\sum_{i \in N} a_i} - \frac{\sum_{i \in N'} a_i \delta_i}{\sum_{i \in N'} a_i}$$ and $$B(\boldsymbol{\delta}) = \frac{\sum_{i \in N} (1 - a_i) \delta_i}{\sum_{i \in N} (1 - a_i)} - \frac{\sum_{i \in N'} (1 - a_i) \delta_i}{\sum_{i \in N'} (1 - a_i)}$$

Rationale

- Compares fraction of **qualified** (or unqualified) persons selected.

Hardt et al. 2016

Possible problem

- Considers only **yes** (or only **no**) decisions.
- **Historical discrimination** can affect who is qualified.

Statistical Fairness Metrics

Predictive rate parity

- Compare $\frac{\text{TP}}{\text{TP} + \text{FP}}$ across 2 groups.

$$B(\boldsymbol{\delta}) = \frac{\sum_{i \in N} a_i \delta_i}{\sum_{\imath \in N} \delta_i} - \frac{\sum_{i \in N'} a_i \delta_i}{\sum_{\imath \in N'} \delta_i}$$

Rationale

- Compares what fraction of selected individuals **should** have been selected.

Dieterich et al. 2016

Problem

- Very difficult nonconvex discrete optimization problem.

Group Parity Metrics

General problems of group parity metrics

- Yes-no outcomes (δ) provide a **limited perspective** on utility consequences (***u***)***.***
- No consensus on **which bias metric** to use (some are mutually incompatible).
- No principle for **balancing** equity and efficiency.
- No clear principle for **selecting protected groups**, unless one simply selects those protected by law.
- Achieving parity for one group may create **disparity for other groups**.

Fairness for the Disadvantaged

Criterion	*Linear?*	*Contin?*
Maximin (Rawlsian)	yes	yes
Leximax (lexicographic)	yes	yes
McLoone index	yes	no

Fairness for the Disadvantaged

Maximin

$$W(\boldsymbol{u}) = \min_i \{u_i\}$$

Rationale:

- Based on **difference principle** of John Rawls.
- Inequality is justified only to the extent that it increases the utility of the worst-off.
- Originally intended only for the design of **social institutions** and distribution of **primary goods** (goods that any rational person would want).
- Social contract argument.

Rawls 1971, 1999

Fairness for the Disadvantaged

Leximax

Rationale:

- Takes in account 2nd worst-off, etc., and avoids wasting utility.
- Can be justified with Rawlsian argument.

Fairness for the Disadvantaged

McLoone index

$$W(\boldsymbol{u}) = \frac{1}{|I(\boldsymbol{u})|\tilde{u}} \sum_{i \in I(\boldsymbol{u})} u_i$$

where $\tilde{u}$ is the median of utilities in $\boldsymbol{u}$ and $I(\boldsymbol{u})$ is the set of indices of utilities at or below the median

Rationale:

- Compares total utility of those at or **below the median** to the utility that would result from bringing them up to the median.
- Index = 1 if no one is below median, $\rightarrow$ 0 for long lower tail.
- Focus on **all** the **disadvantaged**.
- Often used for public goods (e.g., educational benefits).

Utility & Fairness – Convex Combinations

Criterion	*Linear?*	*Contin?*
Utility + Gini coefficient	no	yes
Utility * Gini coefficient	yes	yes
Utility + maximin	yes	yes

Utility & Fairness – Convex Combinations

Utility + Gini coefficient

$$W(\boldsymbol{u}) = (1-\lambda)\sum_i u_i + \lambda\big(1 - G(\boldsymbol{u})\big)$$

Rationale.

- Takes into account both efficiency and equity.
- Can adjust their relative importance.

Problem.

- Combines utility with a dimensionless quantity.
- How to interpret λ, or choose a λ for a given application?

Utility & Fairness – Convex Combinations

Utility * Gini coefficient

$$W(\boldsymbol{u}) = \left(1 - G(\boldsymbol{u})\right) \sum_i u_i$$

Rationale.

Eisenhandler & Tzur 2019

- Gets rid of λ.
- Equivalent to SWF that is easily linearized:

$$W(\boldsymbol{u}) = \sum_i u_i - \frac{1}{n} \sum_{i<j} |u_j - u_i|$$

Problem.

- It is still a convex combination of utility and an equality metric (mean absolute difference).

Utility & Fairness – Convex Combinations

Utility + Gini-weighted utility

$$W(\boldsymbol{u}) = \sum_i u_i + \mu\big(1 - G(\boldsymbol{u})\big) \sum_i u_i$$

Rationale.

- Combines quantities measured in same units.

Mostajabdaveh, Gutjahr, Salman 2019

Problem.

- Equivalent to utility*(1-Gini) with multiplier $\lambda = \mu\,(1 + \mu)^{-1}$.
- How to interpret μ?

Utility & Fairness – Convex Combinations

Utility + Maximin

$$W(\boldsymbol{u}) = (1-\lambda)\sum_i u_i + \lambda \min_i\{u_i\}$$

Rationale.

- Explicitly considers individuals other than worst off.

Problem.

- If u_k is smallest utility, this is simply the linear combination

$$W(\boldsymbol{u}) = u_k + (1-\lambda)\sum_{i\neq k} u_i$$

- How to interpret λ?

Utility & Fairness – Classical Methods

Criterion	*Linear?*	*Contin?*
Alpha fairness	yes	yes
Proportional fairness (Nash bargaining)	yes	yes
Kalai-Smorodinsky bargaining	no	yes

Alpha Fairness

$$W_\alpha(\boldsymbol{u}) = \begin{cases} \dfrac{1}{1-\alpha}\displaystyle\sum_i u_i^{1-\alpha} & \text{for } \alpha \geq 0,\ \alpha \neq 1 \\ \displaystyle\sum_i \log(u_i) & \text{for } \alpha = 1 \end{cases}$$

Mo & Walrand 2000; Verloop, Ayesta & Borst 2010

Rationale.

- Continuous and well-defined adjustment of equity/efficiency tradeoff.

  Utility u_j must be reduced by $(u_j/u_i)^\alpha$ units to compensate for a unit increase in u_i $(< u_j)$ while maintaining constant social welfare.

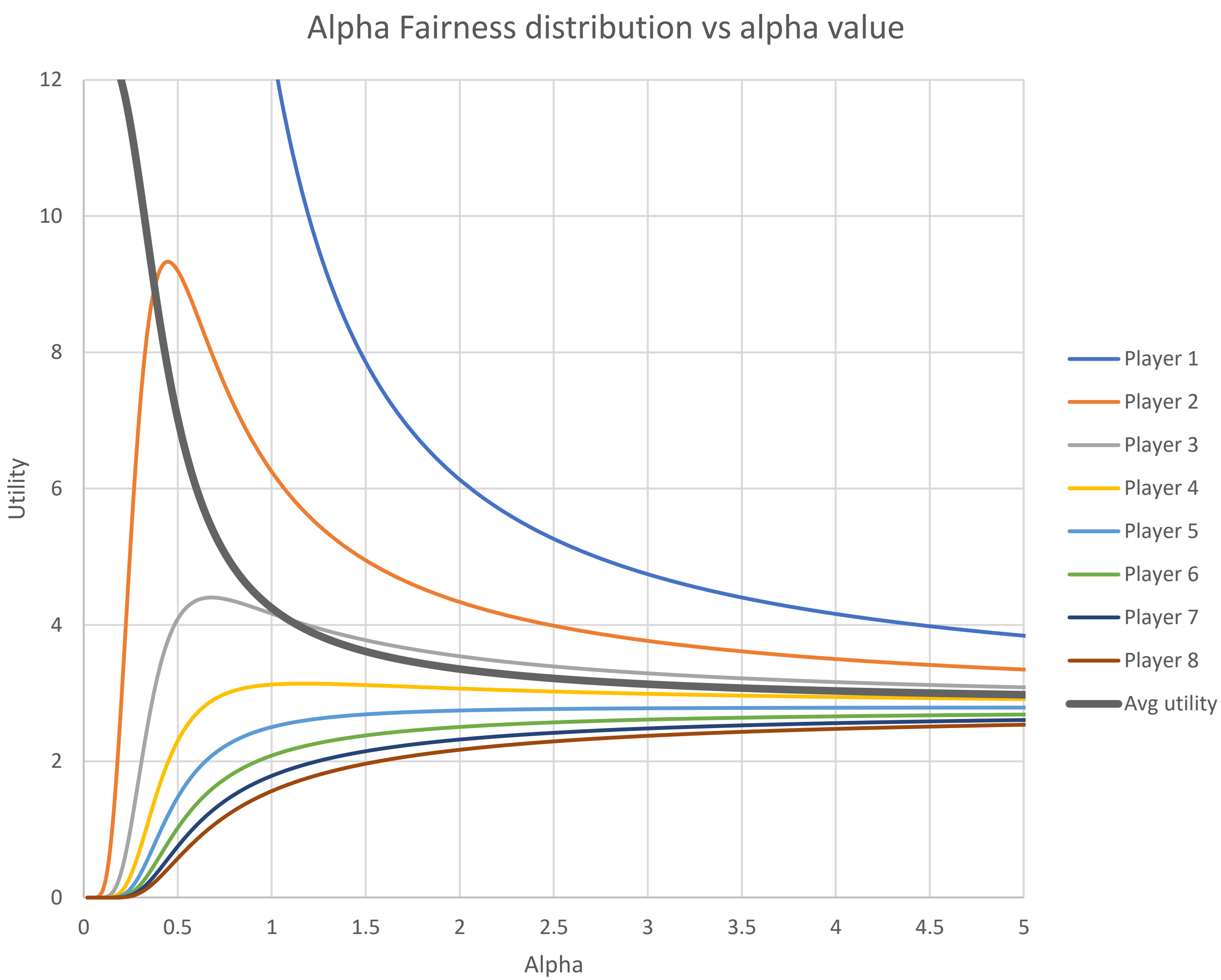
- Utilitarian when $\alpha = 0$, maximin when $\alpha \to \infty$
- Can be **derived from certain axioms**. Lan & Chiang 2011
- Nonlinear but concave

Alpha Fairness

Example:

Maximum alpha fairness subject to budget constraint

$$u_1 + 2u_2 + \cdots + 8u_8 \leq 100$$



Proportional Fairness

$$W(\boldsymbol{u}) = \sum_i \log(u_i)$$

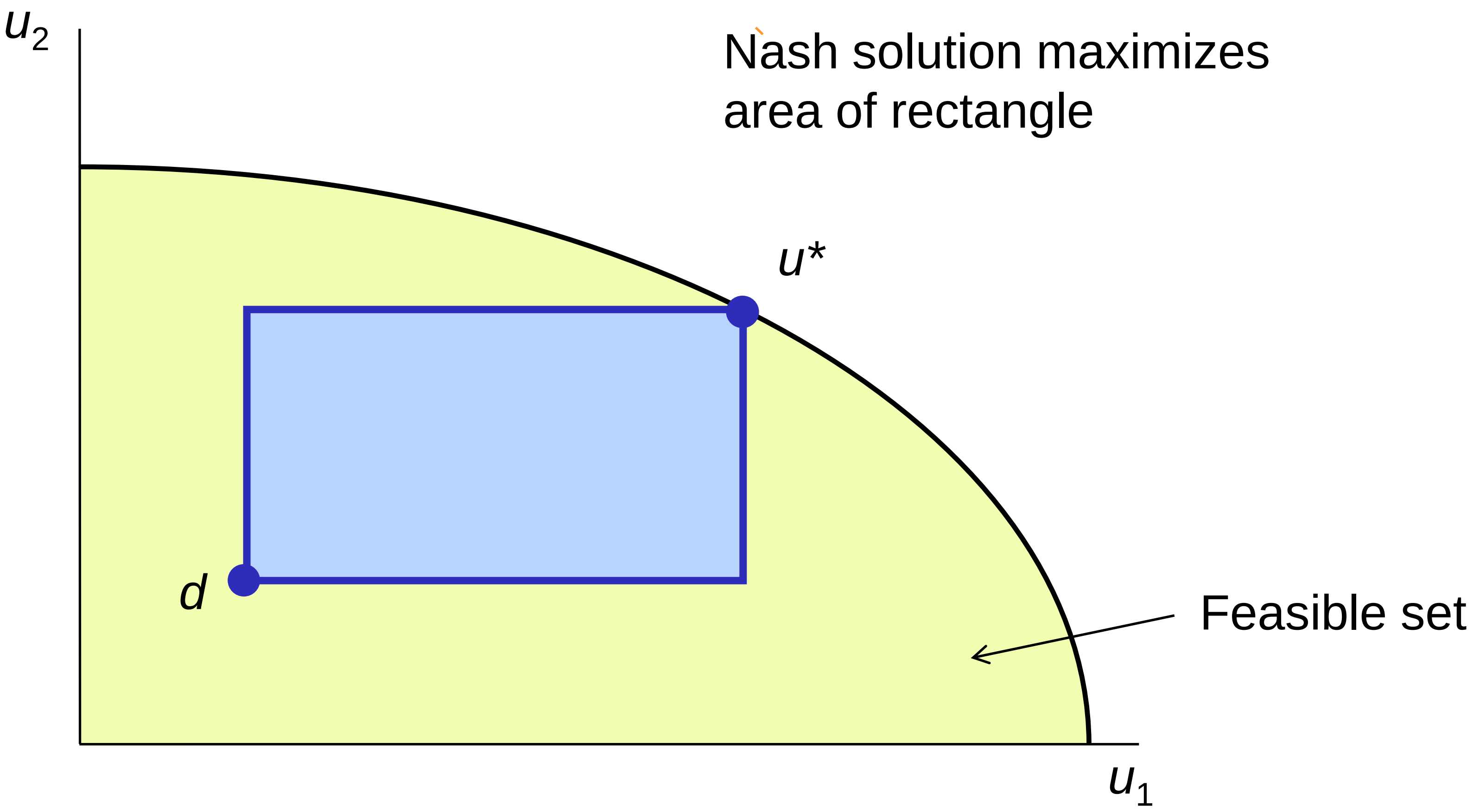
Nash 1950

- Special case of alpha fairness (α = 1).
- Also known as **Nash bargaining solution.**
- Bargaining starts with a default distribution ***d***.

Rationale

- Has nice geometric interpretation.
- Can be derived from axiomatic and bargaining arguments.

Proportional Fairness



Proportional Fairness

Problems with bargaining justifications.

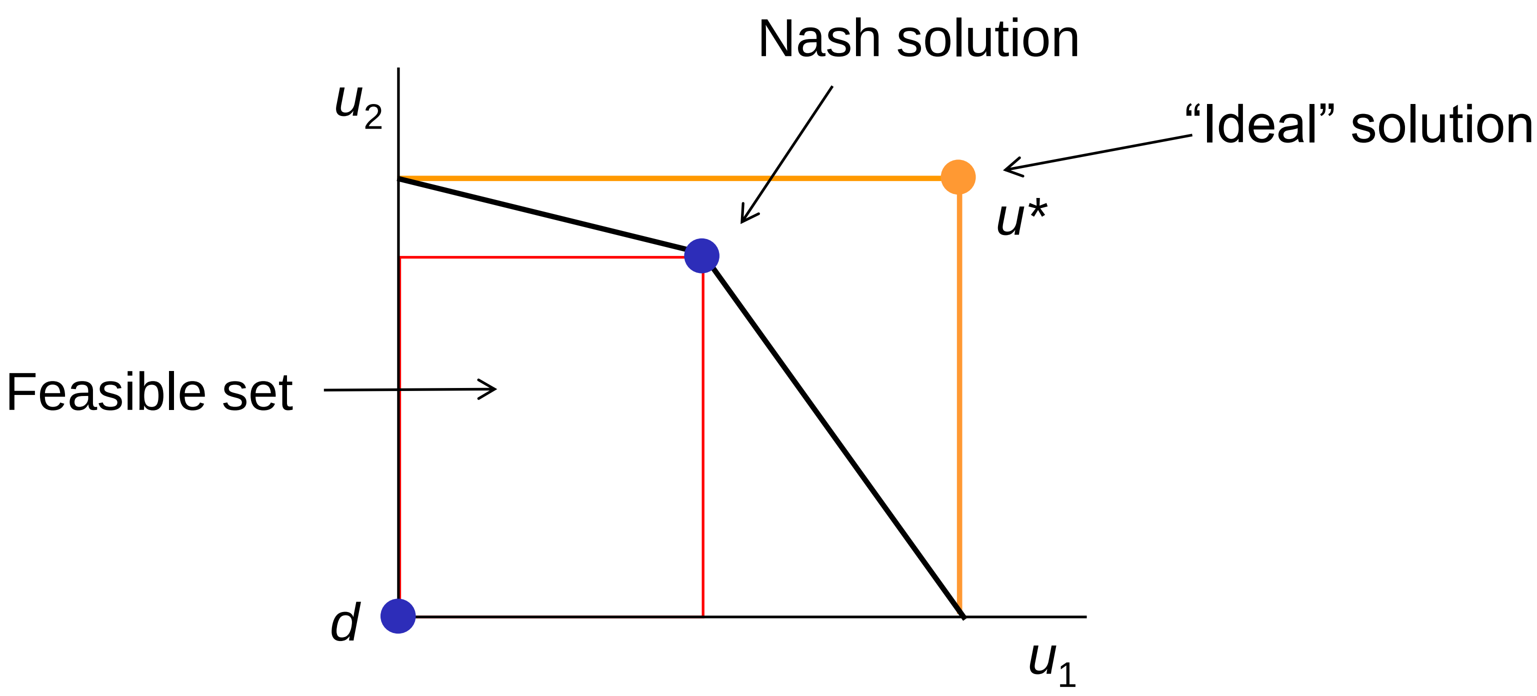
- Why should a bargaining procedure that is rational from an **individual** viewpoint result in a **just distribution?**

- Why should “**procedural justice**” = **justice**? For example, is the outcome of bargaining in a free market necessarily just?

- A deep question in political theory.

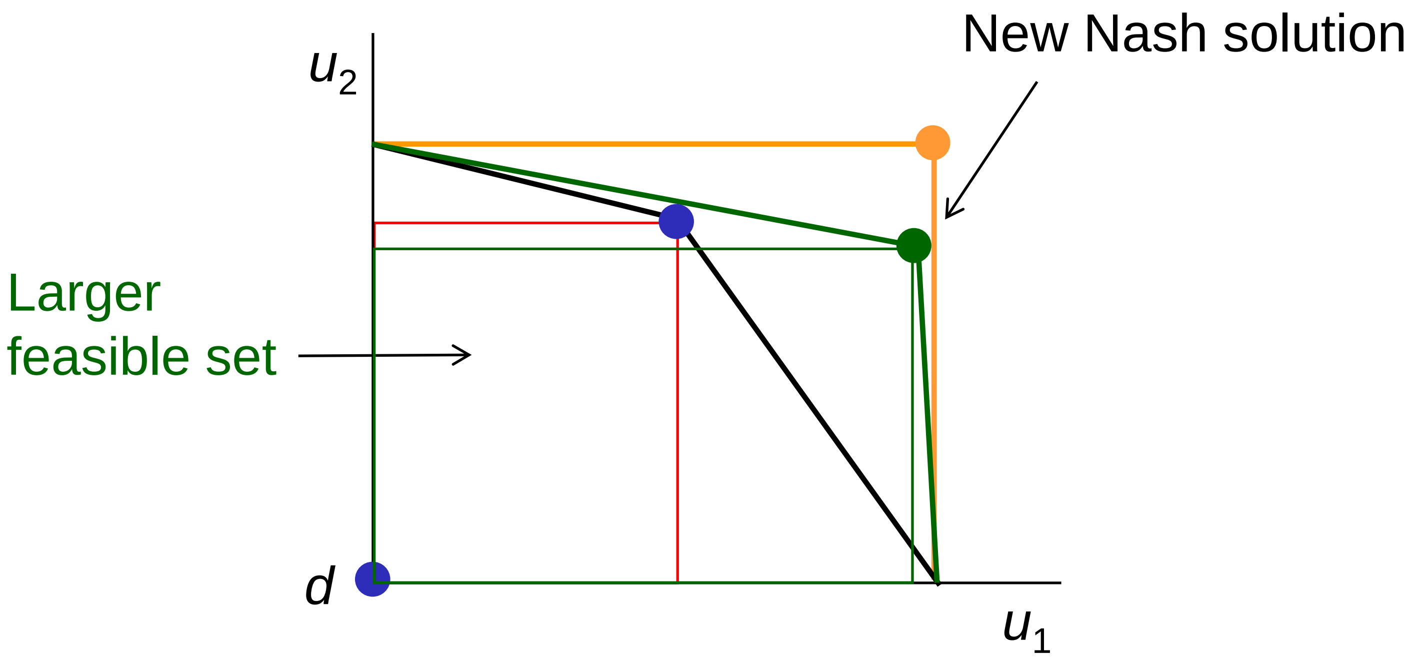
Kalai-Smorodinsky Bargaining

- Begins with a critique of the Nash bargaining solution.



Kalai-Smorodinsky Bargaining

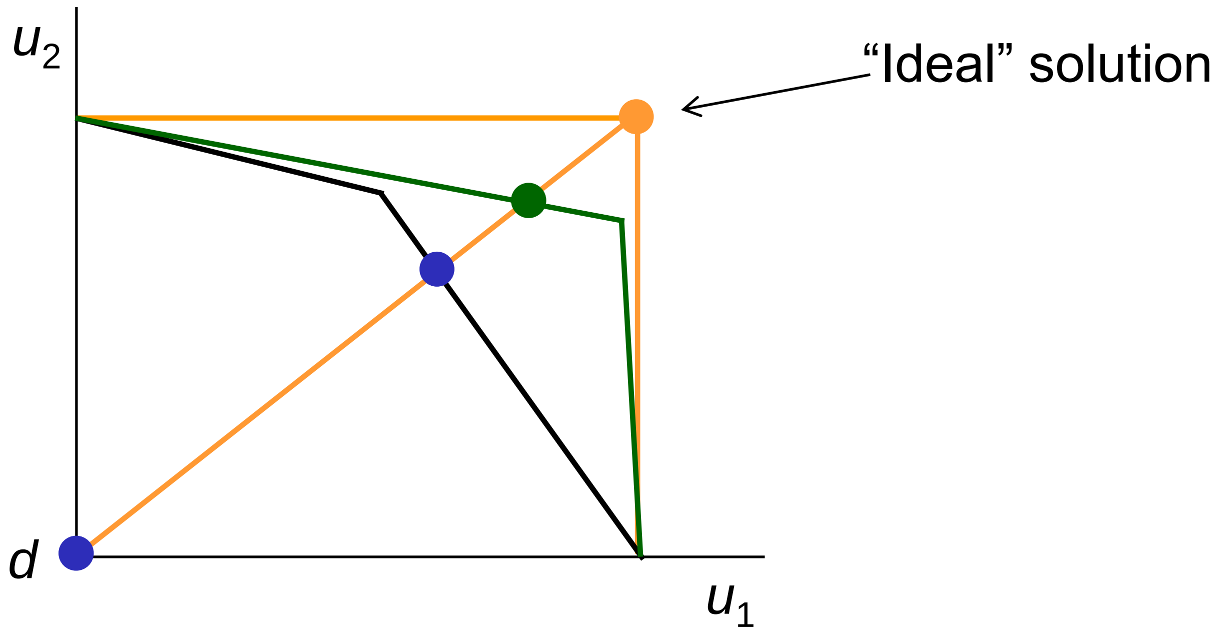
- Begins with a critique of the Nash bargaining solution.
- The new Nash solution is **worse** for player 2 even though the feasible set is **larger**.



Kalai-Smorodinsky Bargaining

- **Proposal**: Bargaining solution is pareto optimal point on line from *d* to ideal solution.

Kalai & Smorodinksy 1975



Kalai-Smorodinsky Bargaining

Possible problems

- In some contexts, it may not be ethical to allocate utility in proportion to one's potential.
- For example, when allocating resources to those with minor ailments vs chronic diseases.

Utility & Fairness – Threshold Methods

Criterion	*Linear?*	*Contin?*
Utility + maximin – Utility threshold	yes	yes
Utility + maximin – Equity threshold	yes	no
Utility + leximax – Predefined priorities	yes	yes
Utility + leximax – No predefined priorities	yes	yes

Threshold Methods

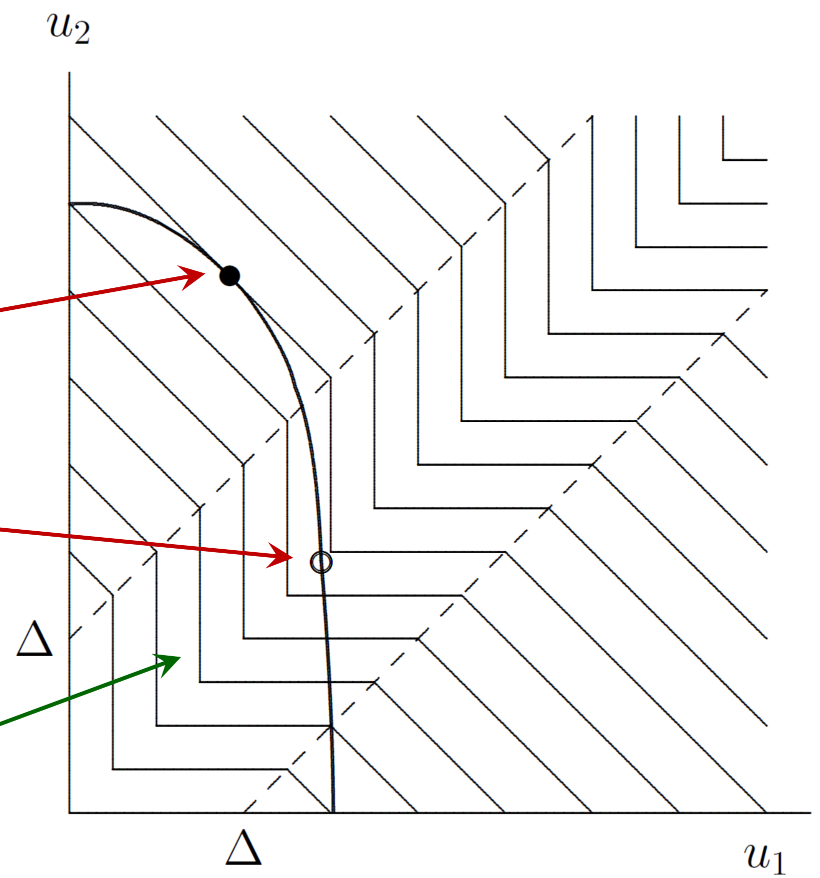
Combining utility and maximin

- **Utility threshold:** Use a maximin criterion until the utility cost becomes too great, then switch a utilitarian criterion.
- **Equity threshold:** Use a utilitarian criterion until the inequity becomes too great, then switch to a maximin criterion.

Williams & Cookson 2000

Threshold Methods

Utility threshold



Optimal solution

Maximin solution results
in too much utility
sacrifice for person 2

Feasible set

Williams & Cookson 2000

$$W(u_1, u_2) = \begin{cases} u_1 + u_2, & \text{if } |u_1 - u_2| \geq \Delta \\ 2\min\{u_1, u_2\} + \Delta, & \text{otherwise} \end{cases}$$

Threshold Methods

Utility threshold

Generalization to *n* persons

$$W(\boldsymbol{u}) = (n-1)\Delta + \sum_{i=1}^{n} \max\left\{u_i - \Delta, u_{\min}\right\}$$

$$\text{where } u_{\min} = \min_i \{u_i\}$$

JH & Williams 2012

Interpretation

- Δ is chosen so that individuals in fair region are sufficiently deprived to **deserve priority**.
- $\Delta = 0$ corresponds to utilitarian criterion, $\Delta = \infty$ to maximin.

Threshold Methods

Utility threshold

Rationale

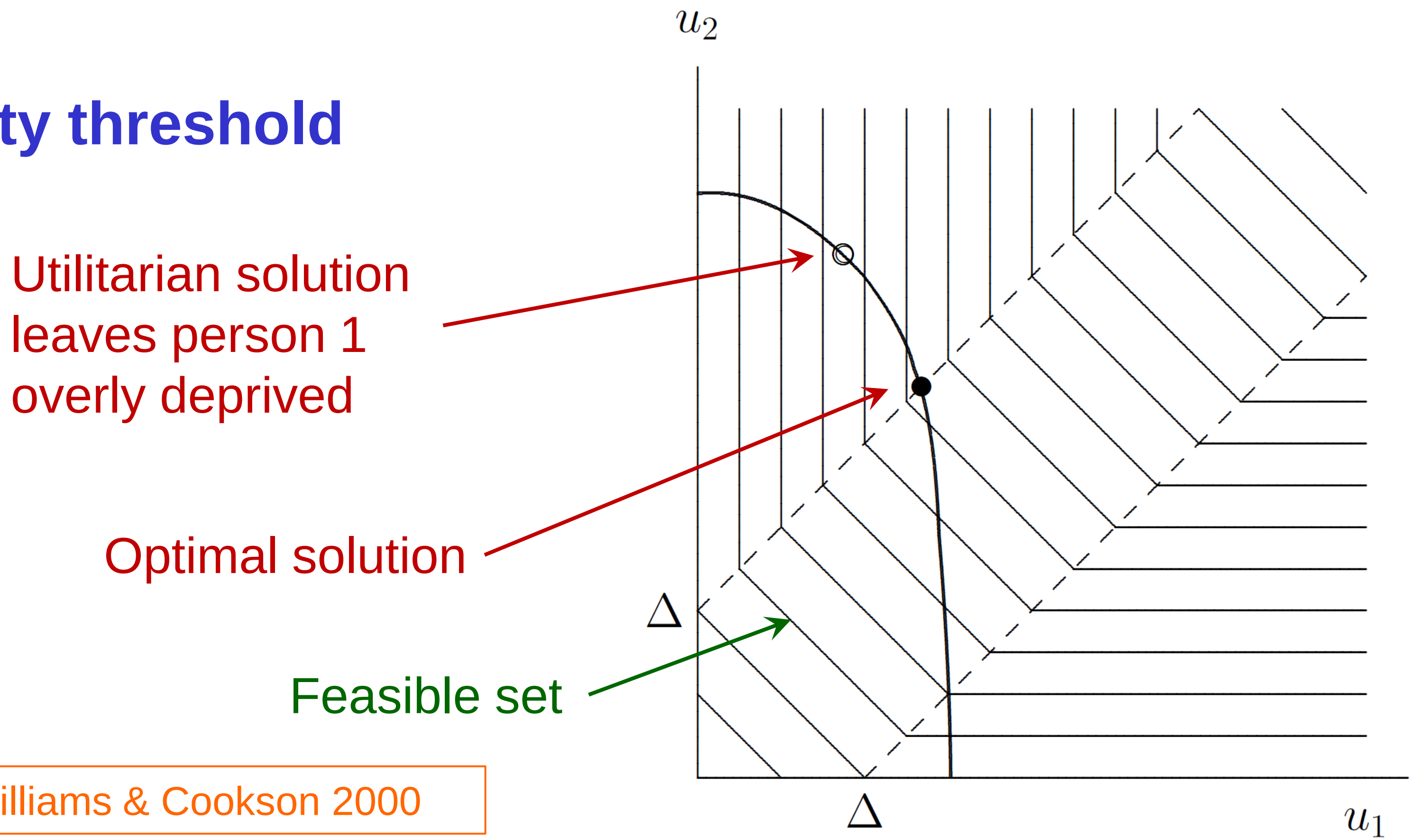
- Trade-off parameter Δ has a **practical interpretation**.
- Suitable when **equity** is the initial concern, but without paying **too high a cost** for fairness (healthcare, politically sensitive contexts).

Problem

- Due to maximin component, many solutions with different equity properties have same social welfare value.

Threshold Methods

Equity threshold



Williams & Cookson 2000

$$W(u_1, u_2) = \begin{cases} 2\min\{u_1, u_2\} + \Delta, & \text{if } |u_1 - u_2| \geq \Delta \\ u_1 + u_2, & \text{otherwise} \end{cases}$$

Threshold Methods

Equity threshold

Generalization to *n* persons

$$W(\boldsymbol{u}) = n\Delta + \sum_{i=1}^{n} \min\{u_i - \Delta, u_{min}\}$$

Elçi, JH, and Zhang 2022

Interpretation

- Δ is chosen so that well-off individuals (those in fair region) **do not deserve more utility** unless smaller utilities are also increased.

Threshold Methods

Equity threshold

Rationale

- Trade-off parameter Δ has a **practical interpretation**.
- Suitable when **efficiency** is the initial concern, but one does not want to create **excessive inequality** (traffic management, telecom, disaster recovery).

Elçi, JH, and Zhang 2022

Problem

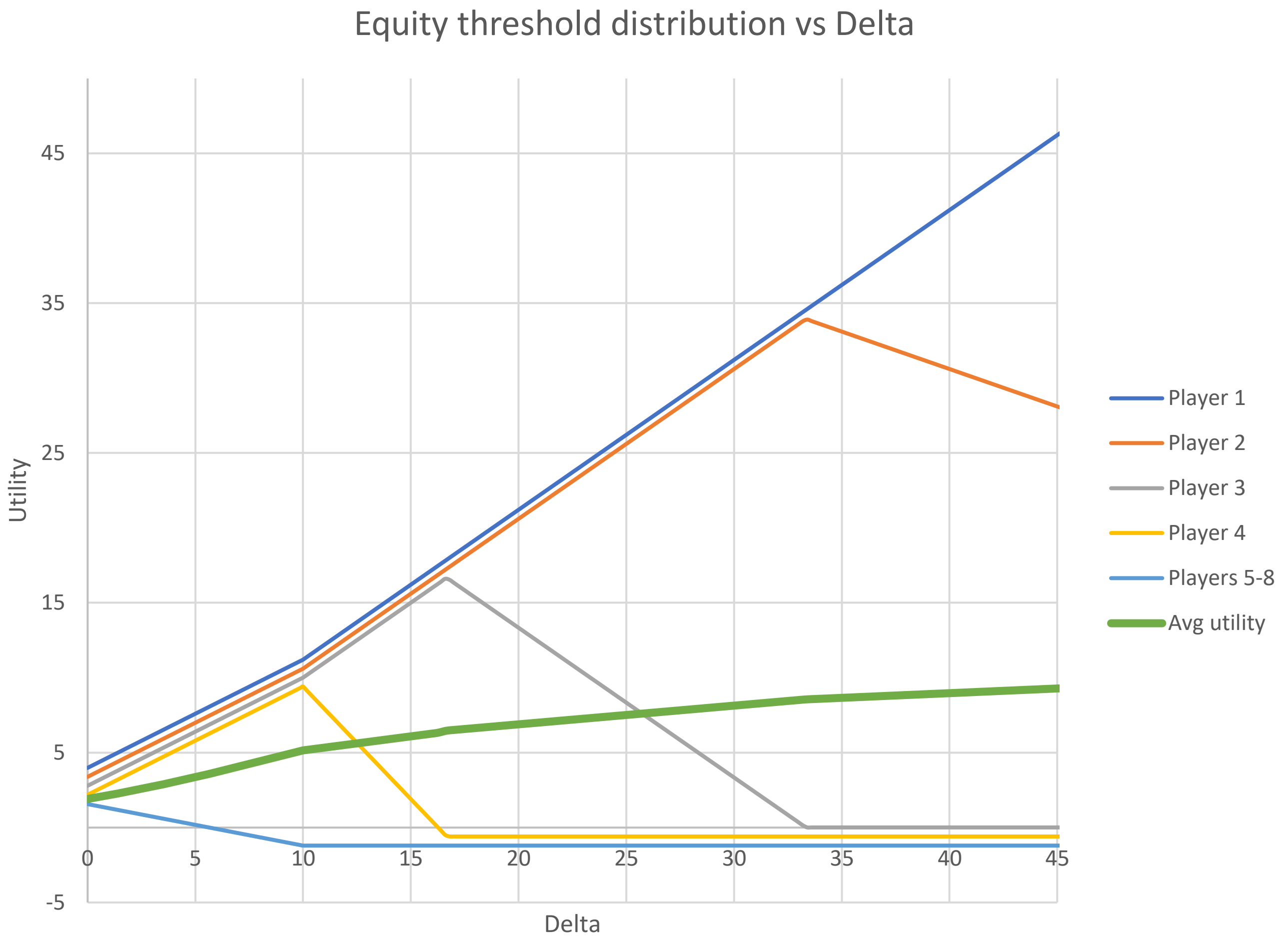
- As with threshold model, many solutions with different equity properties have same social welfare value.

Threshold Methods

Example:

Maximum equity threshold SWF subject to budget constraint

$$u_1 + 2u_2 + \cdots + 8u_8 \leq 100$$



Threshold Methods

Utility + leximax, sequence of SWFs

SWFs $W_1, \ldots, W_n$ are maximized sequentially, where W_1 is the utility threshold SWF defined earlier, and W_k for $k \geq 2$ is

$$W_k(\boldsymbol{u}) = \sum_{i=1}^{k-1}(n-i+1)u_{\langle i \rangle} + (n-k+1)\min\left\{u_{\langle 1 \rangle} + \Delta, u_{\langle k \rangle}\right\} + \sum_{i=k}^{n} \max\left\{0,\ u_{\langle i \rangle} - u_{\langle 1 \rangle} - \Delta\right\}$$

where $u_{\langle 1 \rangle}, \ldots, u_{\langle n \rangle}$ are $u_1, \ldots, u_n$ in nondecreasing order.

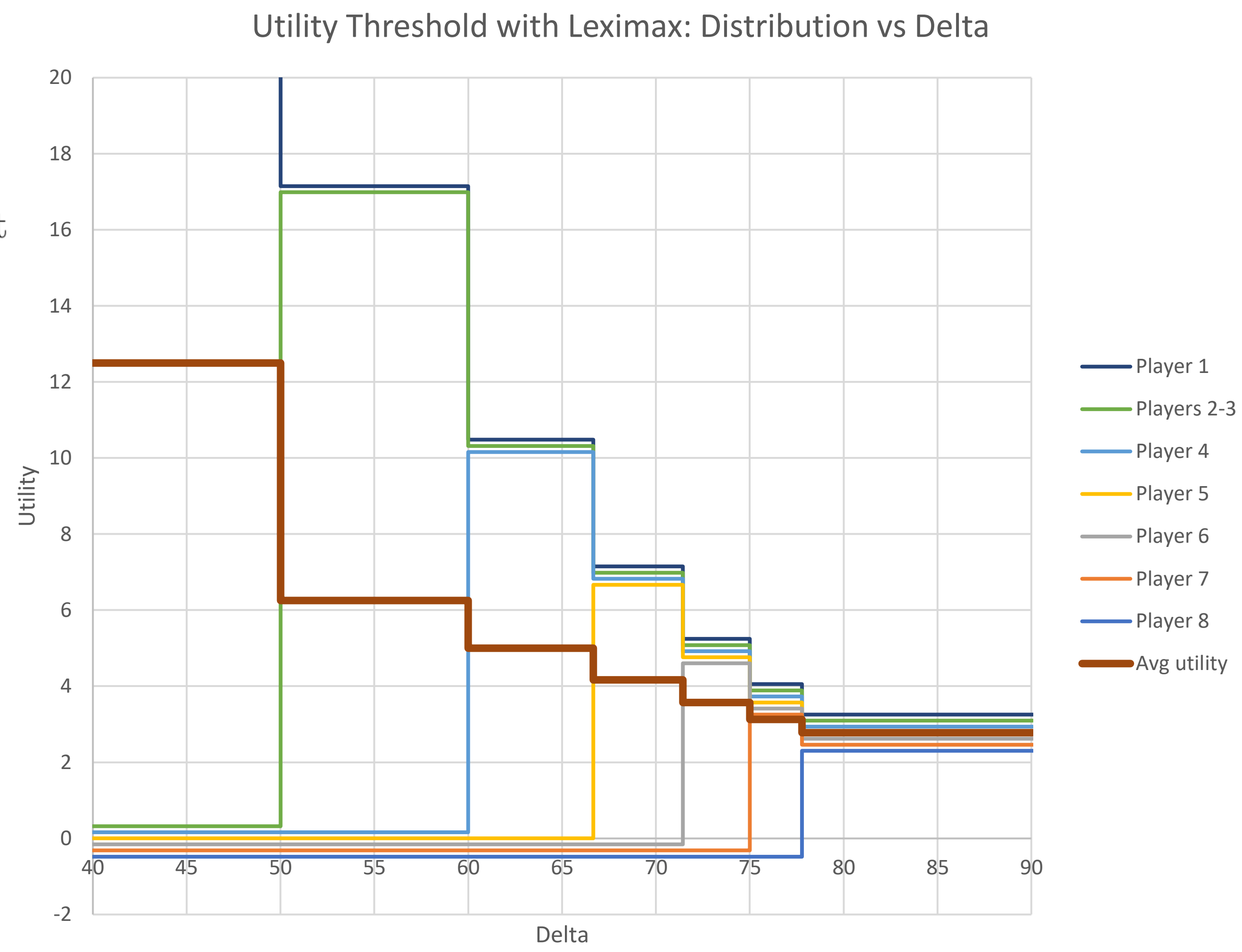
Chen & JH 2021

Threshold Methods

Example:

Maximum utility threshold SWF with leximax subject to budget constraint

$$u_1 + 2u_2 + \cdots + 8u_8 \leq 100$$



Threshold Methods

Rationale

- Takes into account utility levels of all individuals in the fair region.
- Tractable MILP models in practice, valid inequalities known.

Possible problems

- Requires solving a sequence of MILPs.
- Hard to explain and justify on first principles.

Threshold Methods – Healthcare Example

- Based on budget decisions in UK National Health Service
- Allocate limited treatment resources to disease/prognosis categories of patients.
- Based on cost, number of patients, and QALY estimates with and without treatment.*
- We will use a **leximax-utility threshold SWF**.
- Solution time = fraction of second for each value of Δ.

Problem due to JH & Williams 2012

*QALY = quality adjusted life-year. Data reflect a particular situation and are not valid in general. Solutions presented here should not be taken as a general recommendation for healthcare resource allocation, but only as an illustration of social welfare functions.

QALY & cost data

Part 1

Intervention	Cost per person c_i (£)	QALYs gained q_i	Cost per QALY (£)	QALYs without intervention α_i	Subgroup size n_i
Pacemaker for atrioventricular heart block					
Subgroup A	3500	3	1167	13	35
Subgroup B	3500	5	700	10	45
Subgroup C	3500	10	350	5	35
Hip replacement					
Subgroup A	3000	2	1500	3	45
Subgroup B	3000	4	750	4	45
Subgroup C	3000	8	375	5	45
Valve replacement for aortic stenosis					
Subgroup A	4500	3	1500	2.5	20
Subgroup B	4500	5	900	3	20
Subgroup C	4500	10	450	3.5	20
CABG[1] for left main disease					
Mild angina	3000	1.25	2400	4.75	50
Moderate angina	3000	2.25	1333	3.75	55
Severe angina	3000	2.75	1091	3.25	60
CABG for triple vessel disease					
Mild angina	3000	0.5	6000	5.5	50
Moderate angina	3000	1.25	2400	4.75	55
Severe angina	3000	2.25	1333	3.75	60
CABG for double vessel disease					
Mild angina	3000	0.25	12,000	5.75	60
Moderate angina	3000	0.75	4000	5.25	65
Severe angina	3000	1.25	2400	4.75	70

QALY & cost data

Part 2

Intervention	Cost per person c_i (£)	QALYs gained q_i	Cost per QALY (£)	QALYs without intervention α_i	Subgroup size n_i
Heart transplant					
	22,500	4.5	5000	1.1	2
Kidney transplant					
Subgroup A	15,000	4	3750	1	8
Subgroup B	15,000	6	2500	1	8
Kidney dialysis					
Less than 1 year survival					
Subgroup A	5000	0.1	50,000	0.3	8
1-2 years survival					
Subgroup B	12,000	0.4	30,000	0.6	6
2-5 years survival					
Subgroup C	20,000	1.2	16,667	0.5	4
Subgroup D	28,000	1.7	16,471	0.7	4
Subgroup E	36,000	2.3	15,652	0.8	4
5-10 years survival					
Subgroup F	46,000	3.3	13,939	0.6	3
Subgroup G	56,000	3.9	14,359	0.8	2
Subgroup H	66,000	4.7	14,043	0.9	2
Subgroup I	77,000	5.4	14,259	1.1	2
At least 10 years survival					
Subgroup J	88,000	6.5	13,538	0.9	2
Subgroup K	100,000	7.4	13,514	1.0	1
Subgroup L	111,000	8.2	13,537	1.2	1

Threshold Methods – Healthcare Example

Budget constraint

$$\sum_j n_j c_j x_j \leq B$$

- n_j: Size of treatment group *j*
- c_j: Unit cost of treatment *j*
- x_j: Fraction of group treated

Utility function

$$u_i = q_i x_i + \alpha_i$$

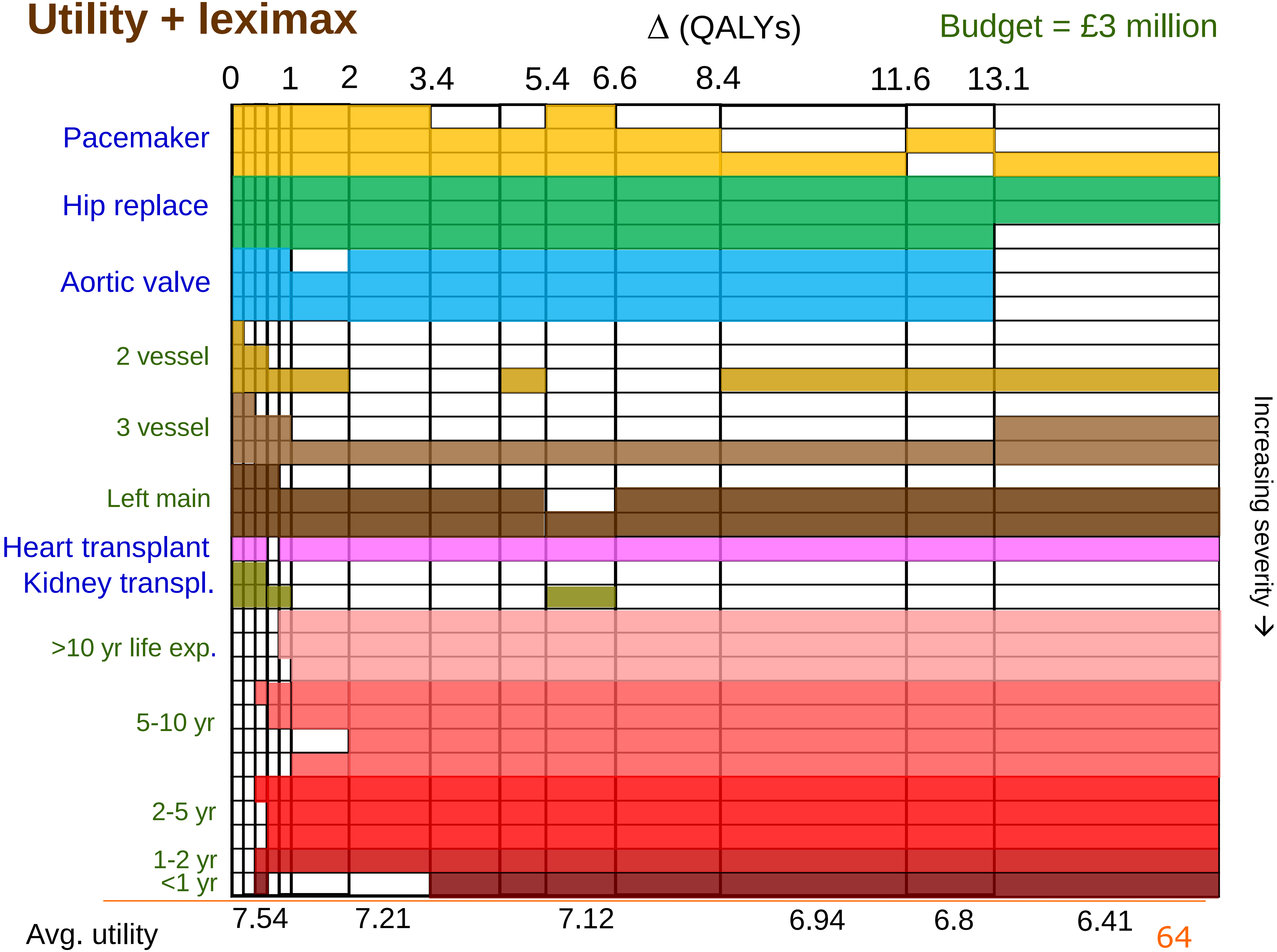
- q_i: Treatment benefit (QALYs)
- α_i: QALYs without treatment

which implies $x_i = (u_i - \alpha_i)/q_i$

So the optimization problem becomes

$$\max_{\boldsymbol{u}} \left\{ W(\boldsymbol{u}) \;\middle|\; \sum_j \frac{n_j c_j}{q_j} u_j \leq B + \sum_j \frac{n_j c_j \alpha_j}{q_j};\;\; \boldsymbol{\alpha} \leq \boldsymbol{u} \leq \boldsymbol{q} + \boldsymbol{\alpha} \right\}$$

Utility + leximax



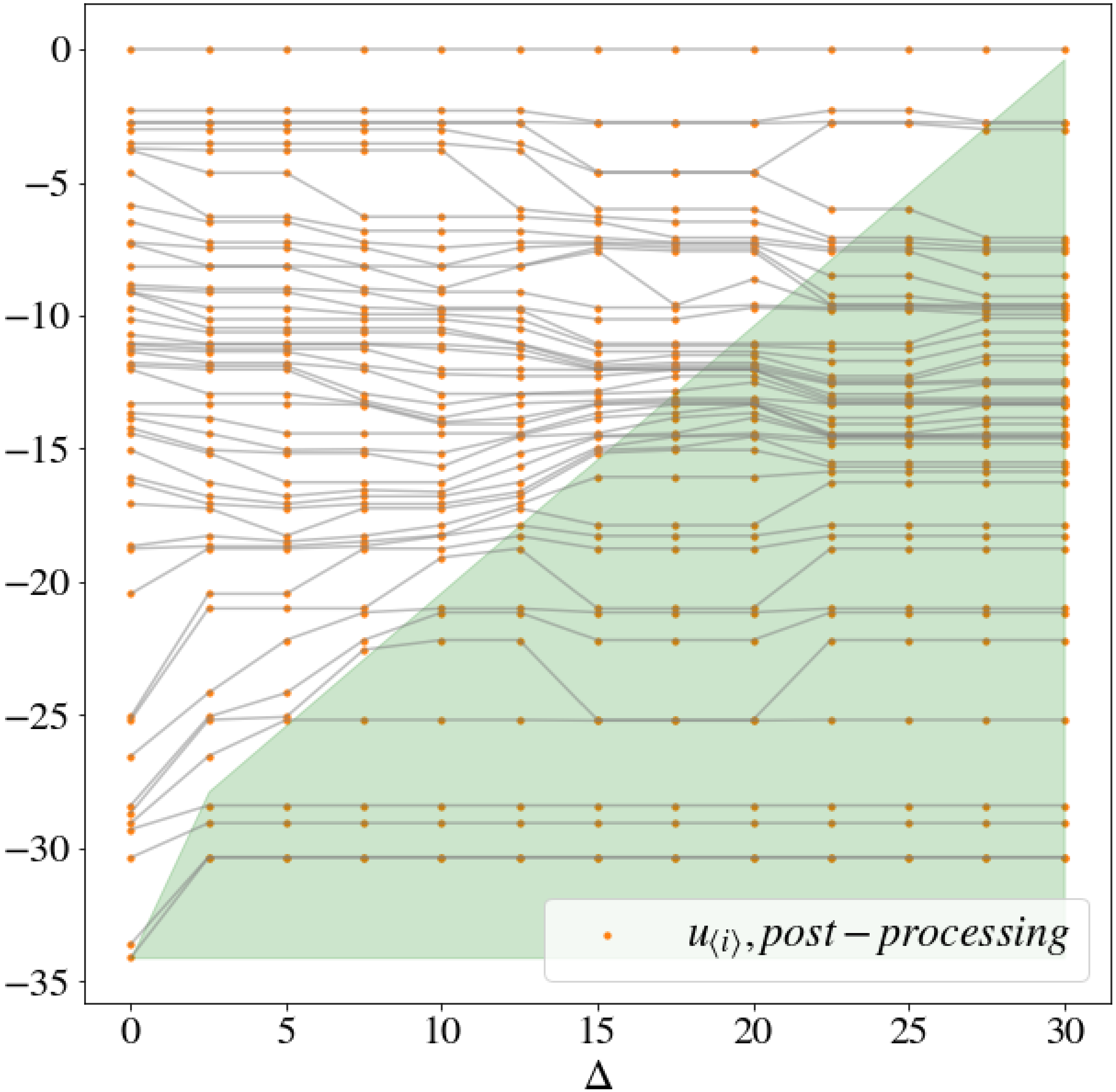
Threshold Methods – Disaster Preparedness Example

- Select earthquake shelter locations.
- Utility = negative distance of each neighborhood to nearest shelter, subject to limited budget.
- We will use a **leximax-utility SWFs**.
- 50 neighborhoods, 50 potential shelter locations.
- Solution time = 1 to 18 seconds for each value of Δ.

Problem due to Mostajabdaveh, Gutjahr & Salman 2019

Threshold SWF

Utility + leximax




Questions or comments?